Journal of Didactic Mathematics
2022, 3(3), 94-109
Doi: 10.34007/jdm.v3i3.1569
https://mahesainstitute.web.id/ojs2/index.php/jdm
journalofdidacticmathematics@gmail.com

Thinking about the equal sign: What do students see about the equal sign?

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Abstract. The equal sign seems to be interpreted differently by students depending on the learning experiences they have had in the early grades. In fact, the interpretation of the equal sign as a relational symbol does not seem easy or fast for many students to understand. This study aims to explore how students from elementary school to college students describe their understanding of the equal sign. The Qualitative Comparative Analysis can be used to analyze several cases in complex situations so that it fits the purpose of this study. The process of collecting data through the method of written assignments, semi-structured interviews, and documentation was carried out in one time period. This study involved 30 participants in Bandung, Indonesia. The results show that although there are substantial differences in viewing the equal sign, there are similarities in terms of dependence on computational methods in drawing conclusions. This is related to how their experience of number sentences in lower grades places more emphasis on rules than on the meaning of concepts.

Article History:
Received: November 14, 2022
Revised: December 14, 2022
Accepted: December 30, 2022

Keywords:
Early algebra; operational; relational; the equal sign

Citation: Ardiansari, L., Suryadi, D., & Dasari, D. (2922). Thinking about the equal sign: What do students see about the equal sign?. *Journal of Didactic Mathematics*, *3*(3), 94-109. Doi: 10.34007/jdm.v3i3.1569

INTRODUCTION

Students' understanding of the equal sign is one of the important foundations for their success in learning mathematics because there is no branch of mathematics that does not use it (Carpenter et al., 2003; McNeil et al., 2006; Stephens et al., 2013). This understanding involves subjective mental activity so that each individual can have a different meaning according to their learning experience (Kieran, 1981; McNeil et al., 2006; Sherman & Bisanz, 2009). The understanding that students gain from previous experiences can be entrenched and consistent over the long term (Knuth et al., 2006). Knowledge gained from initial experience if it does not match the information found in future efforts, it will have a 'top-down' effect so that early learning difficulties become the cause of later learning difficulties (Bruner, 1957; Rumelhart, 1980). So that students' misunderstanding in interpreting the meaning of the equal sign in early grades can be a cause of learning difficulties later in life even at the higher education level (Baiduri 2015; Best, McRoberts, & Goodell, 2001; Flege, Yeni-Komshian, & Liu, 1999; Knuth et al., 2006).

Although the equal sign is a very important concept for understanding mathematics at all levels of school mathematics, the meaning of 'equal' is a complex and difficult idea for most students to understand (Alibali, 2007; Kieran, 1981). Much thought and research in various countries has explored learners' understanding of the equal sign in primary schools (Carpenter, Levi, Franke & Zeringue, 2005; Darr, 2003; Fuchs et al., 2014; Hattikudur, & Alibali, 2010; Vermeulen & Meyer, 2017), secondary schools (Alibali et al., 2007; Kindrat & Osana, 2018;

Kiziltoprak & Kose, 2017; McNeil et al., 2006; Solares & Kieran, 2013), and universities (Baiduri, 2015; Stephens, 2006). This shows that understanding the equal sign has become the focal point of reform and research efforts in mathematics education as a proven strong and long-term 'problem' that can be experienced by elementary, middle school and even college students. Although this misconception has been studied for more than thirty years in various countries, it has not provided much refinement in theory (Capraro, 2007). Thus, the meaning of the same sign cannot be underestimated and even has a high urgency to be discussed thoroughly.

There have been many studies on students' understanding of the meaning of the equal sign, but there are still not many thorough discussions in mathematics education about the strong meaning of the equal sign and have not carried out a thorough exploration of the meaning of the equal sign based on students' learning experiences at various levels (for example, Baiduri, 2015; Kindrat & Osana, 2018; Vermeulen & Meyer, 2017). Of the previously mentioned studies, none have used the methodology of qualitative comparative analysis to help explain why change occurs in some cases but not in others. This study, will identify the meaning of the equal sign with students based on the results of their school's arithmetic and algebra learning experience. If the meaning of the sign is the same as that understood by students deviating from the true meaning of mathematics, then from the results of this study it is hoped that the experience of what, why and how became the beginning of the deviation. Therefore, the purpose of this study is to analyze and explore various qualitative comparative aspects in understanding the equal sign in three parts: performance, strategies, and learning experiences used in completing written assignments. So that basic things can be identified to provide a more meaningful learning experience since elementary school, understanding the concept of the same sign as approaching the real mathematical meaning, and supporting relational thinking.

The meaning of the equal sign with '=' (the equal sign) in mathematics expresses identity relations, namely, the same 'is equal to', 'is the same as' or identical to 'is identical to' where all refer to the specific relationship of the equivalence relation. That is, equality has a different meaning from equivalence because there are other equivalence relations that do not refer to the same relationship. Although there are differences, equivalency and identity do have some of the same characteristics, namely identity (equality) is a certain relationship of equality in the reflective, transitive, and symmetrical sense of Mirin (2019). Furthermore, Mirin stated that in order to equate equality and similarity, Galois' theory must first be considered, which involves calculating isomorphism.

A weak understanding of the same relationship is described by the word 'operational', namely seeing the equal sign involving the performance of an operation (Mirin, 2019). In some literature, many authors give the 'operational' view as a false or unproductive understanding and contrast with the relational view. This relational (non-operational) view involves looking at the equal sign denoting an equivalence relationship (Carpenter, Franke, & Levi, 2003; Kieran, 1981); the relationship between the two quantities (Knuth et al., 2008); the relationship between the same values of two numerical expressions (Oksuz, 2007; Sáens-Ludlow & Walgamuth, 1998). According to Mirin (2019), seeing the equal sign as an equivalence relationship is responsible for the way of thinking where, for example (i) $5 = 2 + 3$, (ii) $2 + 3 = 4 + 1$, and (iii) $5 = 5$ is not a violation. rules, but rather the ability to be replaced. Let '=' be an equivalence relation. As long as $2 + 3 = 5$, then from symmetry $5 = 2 + 3$. Similarly, it follows from reflexivity that $5 = 5$. Thus (i) and (iii) are no longer a rule violation. Conclusion (ii) can be easily obtained through symmetry and transitivity, i.e., $2 + 3 = 5$, with symmetry obtained $5 = 4 + 1$. Therefore, it follows from transitivity that $2 + 3 = 4 + 1$.

Throughout elementary school, students have experience in perceptual patterns of arithmetic problems (McNeil & Alibali, 2004). The equal sign and empty answers in arithmetic problems are usually presented regularly at the end of problems such as 4 - 3 + 6 =____. However, many higher level problems do not have a blank answer at the end of the problem. Students' narrow experience with arithmetic in elementary school reinforces representations, strategies and concepts that are not easily transferred beyond traditional arithmetic such as operations on the left side followed by an equal sign and blank answers and signals to count all numbers as 'totals' (McNeil & Alibali, 2004, 2005; McNeil et al., 2017). Excessive practice with arithmetic operations hinders subsequent learning of more complex equations (McNeil & Alibali, 2005) because children only develop 'routine skills' with arithmetic operations, but do not understand the concepts underlying what has been learned (Hatano, 1988; McNeil & Alibali, 2005). Thus, knowledge of perceptual patterns cannot help (and may hurt) student performance. This can have a 'top-down' effect where knowledge gained from initial experience does not match information found in future endeavours (Bruner, 1957; McNeil & Alibali, 2005; Rumelhart, 1980), hence the probability of error increases. This effect relates to the 'reinforcement' of patterns that individuals learn based on experience that becomes entrenched, and then, new information that overlaps with those patterns is assimilated into those patterns. This is beneficial when information fits a learned pattern but can lead to errors when it does not (Best, McRoberts, & Goodell, 2001; Flege, Yeni-Komshian, & Liu, 1999).

The transition period occurs in junior high school students, they are at a stage between the 'equals sign' as a computational sign and receiving the equal sign as a symbol of equality. One of the students' difficulties in the transition period is giving meaning to mathematical symbolism using 'word problems'. The use of 'arithmetic identity' for the term 'equation' to be used in an algebraic sense can broaden the notion of the equal sign thereby providing a basis for further meaning construction for algebraic equations (Kieran, 1981). The use of meaningful symbols and conscious manipulation of algebraic ideas are the two basic elements of algebraic reasoning at the middle-class level that are directly related to understanding the equal sign (Harbour, Karp, & Lingo, 2017). One of the causes of students' difficulties in learning algebra is a paradigm shift from equations. The equal sign in arithmetic is interpreted as 'to give' or 'to produce'. Meanwhile, when introduced to algebra, the equal sign means 'equivalent' so it no longer means 'result' or 'gives' (Schliemann et al., 1998).

There is no evidence to suggest that high school and college students change in their thinking about seeing the equal sign as a 'sign to do something' rather than as an equality sign. Therefore, students' conception of the equal sign should be developed from the equal sign in elementary school and develop into relational understanding in secondary school to support better algebraic competencies, including skills in solving equations and algebraic reasoning (Alibali et al., 2007; Baiduri, 2015; Kieran, 1981; Knuth et al., 2006). Students need a relational perspective in learning to solve algebraic equations using operations on both sides for example, $5x - 5 = 2x + 1$ and understand that the transformations made in the process of solving equations must maintain equality (Baiduri, 2015).

METHOD

This study uses a qualitative comparative analysis (QCA) methodology, a case-based approach that is regularly used to investigate situations in specific contexts and settings. QCA seeks to generate findings across multiple case studies, addressing the complexity and influence of context. It is based on two assumptions: first that change is often the result of a combination of factors, not on a single individual factor; and second, that the combination of various factors can produce similar changes (Rihoux & Ragin, 2008). The qualitative approach was chosen because this study aims to investigate things that exist in the natural environment (natural settings) and try to interpret these phenomena. The comparative perspective is considered the most suitable for the purpose of this study because this study was conducted on three groups of students with different grades. The reliability techniques used in this study include field notes, audio recorders for accuracy, and coding techniques. Themes and codes are derived from qualitative data informed by a theoretical framework that supports research and literature review. Validation techniques identified as commonly used in this study included triangulation of data sources to corroborate evidence, and member checks were performed to determine the credibility of findings and interpretations.

Sampling and Participants

The sample selection used in this study used an incidental sampling technique involved selecting individuals who happened to be available and accessible at the time. Researchers take samples arbitrarily (whenever and wherever they find) as long as they meet the requirements as samples from the desired population, so that bias can be minimized. In this case, the researcher does not attempt to generalize about the wider population.

There were 30 participants in this study. All participants were elementary school students aged 12 to graduate students aged 45 years, had studied the equal sign in arithmetic and/or in school algebra, and were willing to voluntarily become participants. The respondents of this study consisted of 30% men and 70% women. There are 20% primary school students, 37% secondary school students, 43% college students in Bandung, Indonesia. The secondary school group consists of junior and senior high school students, while the college students group consists of undergraduate and postgraduate students in Bandung, Indonesia.

Before being asked for personal consent by the researcher, students have been given an understanding that they will be given the task of answering several questions about the equal sign, answers must be honest according to their personal thoughts and any answers they give are not related to their school grades, and these answers will be documented in a research report with guaranteed anonymity. The researcher assured that the participants that the data collected will be treated confidentially and that no punitive action will be taken against them if they decide to withdraw. All participants have filled out a written statement that they are willing to participate in this study voluntarily and without coercion from any party. Students who are willing to complete the task are the participants of this study.

Instrument

The instrument used in this study was a task-based interview referring to Alibali et al. (2007); Carpenter, Franke & Levi (2003); Kiziltoprak & Kose (2017); Knuth et al. (2006, 2008); Matthews et al (2010); and Rittle-Johnson et al. (2010) were used to describe participants' understanding of the equal sign and the strategies they adopted in solving equivalent equations. All questions in assignments and interviews are in Indonesian. The tasks given consist of two types, namely true or false number sentence questions and open number sentence questions covering each of the following four levels in order to increase the difficulty.

a) Rigid operational, where students are asked to solve problems in the standard "a + b = c" format;
b) Flexible operational, where students are asked to solve equations in several non-standard formats such as “c = a + b”;
c) Relational with computational support, allows students to solve equations with operations on both sides such as “a + b + c = a + _____”;
d) Relational without a need to compute, where the relational view predominates and children demonstrate an understanding of the arithmetical nature of equivalence.

There are four main types of questions that are used in several arithmetic and symbolic formats, namely: type 1 (arithmetic identity) which aims to interpret the equal sign and provide answers; type 2 (algebraic equations) aims to determine solutions to algebraic equations; type 3 (mathematical statement) to interpret the equal sign by stating whether the number sentence is true or false and write the mathematical sentence into ordinary sentences; and type 4 (symbol interpretation) to interpret the meaning of the symbols “=, _____ , $\square$, $\triangle$, and n“. After the respondent completes the written task then proceed with the interview process. Investigations and follow-up questions were added as needed to encourage elaboration and clarification of responses.

Specific questions were added as the interview process progressed according to the responses given by the informants.

Research Procedure

The data collection process is carried out in one period of time. Each respondent was asked to fill in the questions from the written assignment given for 30 minutes. Each respondent was allowed to ask the researcher about anything they did not understand from the assignment and was allowed to use calculating tools such as calculators if they needed to. After they filled out the written task, the researcher then conducted an interview session with a duration of between 10 to 15 minutes. Each question asked aims to confirm their answers, difficulties or obstacles they may experience, what experiences, and how so that they have an understanding of the sign of the same meaning, contextual questions, as well as additional questions according to the responses given. The documentation carried out included students' written answers along with every scribble they wrote on the answer sheets provided, audio recordings during the interview process, and analysis of textbooks used in schools.

Data Analysis

The data that has been collected was analysed using the Miles and Hubberman interactive analysis model, namely through (1) data reduction: sorting the data obtained to be used as research reports, (2) presenting data: grouping or classifying data and selecting according to their type, and (3) data interpretation: interpret what has been given by the informant to the problem under study. Data collected from answers to written assignments were analysed using the following categories: (a) correct and incorrect responses; and (b) operational and non-operational arguments. Incorrect responses were assigned a value of zero (0) and correct responses were assigned a value of one (1). Both incorrect and correct responses, related to understanding the equal sign, were followed up during the interview. Responses were coded as operational if a student expressed the general idea that an equal sign means 'add numbers' or 'answer', and 'signal count' (Alibali et al., 2007) and coded as non-operational if without calculations or with calculations only. as a means of justifying written relational responses (Kindrat & Osana, 2018). The data from the interviews were written as is into a transcript and coded for each respondent. After each transcript was read several times, the researcher highlighted the key concepts and matched them with the answers on the written assignment. In addition, the reduction and elimination of statements that are not experiential horizons are carried out to understand the experience and whether the statements can be abstracted and labelled.

RESULTS AND DISCUSSION

The findings of this study will be presented and discussed according to the theme of the research questions, namely: student performance, strategies used, and learning experiences applied in completing written assignments.

Student Performance

The performance of all respondents in completing assignments is presented in Table 1. Table 1 shows that respondents perform better on tasks that involve computation than those that do not. This is one of the important results of this study, namely that respondents both a number of students and a number of adults responded to open number sentences and true/false number sentences based on a result-oriented process so that they had a 'must to count'. Students were more likely not to perceive the equal sign as a symbol indicating a relationship but as a 'find a result or do something' (Barlow & Harmon, 2012; McNeil et al., 2006; Stephens et al., 2013), a 'sign to count' (Kieran, 1992; Seo & Ginsburg 2003), and 'operation symbols or symbol-syntactic indicators used before answers' (Kieran, 1981; Warren, 2006).

Table 1. The student performance per grade & tasks

Grade	Task 1		Task 2		Task 3		Task 4	
	C (%)	IC (%)	C (%)	IC (%)	C (%)	IC (%)	C (%)	IC (%)
Primary	17%	83%	0%	100%	0%	100%	0%	100%
Scondary	79%	21%	50%	50%	54%	46%	53%	47%
College	90%	10%	86%	14%	85%	15%	82%	18%

Note: (C) = Correct, (IC) = Incorrect

This result is in line with several previous studies that this 'must to count' often encourages the operational view of the equal sign (Baroody & Ginsburg, 1983; Borenson, 2013; Carpenter, Franke, & Levi, 2003; Darr, 2003; Kiziltoprak & Kose, 2003; Machaba, 2017; McNeil, 2008). The operational view of the equal sign shows limited knowledge of the meaning of the equal sign (Knuth et al., 2008; Matthews, 2010; Mirin, 2019) which can hinder students' performance in algebra (Alibali, 2007; Carpenter, Franke, & Levi, 2003; Knuth et al., 2005; Leavy, Hourigan, & McMahon, 2013). The operational view of the equal sign is largely claimed by some researchers as a 'side effect' of students' experiences with symbols in elementary school mathematics (Baroody & Ginsburg, 1983; Carpenter et al., 2003; Knuth et al., 2008; McNeil & Alibali, 2005). Table 2 provides a specific description of the performance of primary school students for each question.

Elementary school students still have an immature working memory system, lack of processing speed, less proficient with basic arithmetic operations, and learning experiences that are still limited to the context of the equal sign as showing the results. Children only develop 'routine skills' with arithmetic operations but do not understand the concepts underlying what has been learned (Hatano, 1988). In task 1, although it is only oriented to arithmetic operations in standard format, the performance of all students is not optimal because they are used to the right side of the equal sign showing the result. Students tend to interpret expressions in the same way as reading numbers, namely sequentially from left to right (Gunnarsson, Sonnerhed, & Hernell, 2016). For example, children who fill in 2 as an answer in the first question, think that the sign of subtraction is the difference between two numbers without paying attention to where the equal sign is.

Task 2 has a flexible operating difficulty level where students are asked to solve equations in several non-standard formats. This type of question posed in task 2 allows respondents to induce equivalent pairs of adjuncts based on a transitive relationship. This ability is a fundamental arithmetic skill that allows writing number sentences with mathematical symbols, understanding the basic features of operations and conceptualizing numbers in various forms (Kiziltoprak & Kose, 2017). All students in this grade are very difficult to give the expected response. In fact, in questions 1 and 2 there were two students who agreed to answer 4 because "there is a box, the box has 4 sides, so the answer must be 4" (Respondent 1, May 2019: personal interview) without paying attention to the mathematical sentence at all.

In tasks 3 and 4, all elementary school respondents gave up entirely because they had never encountered a similar problem before. It appears that elementary school curriculum materials often present the same sign in contexts that support operational interpretation, such as 'operation equals answer' problem structures (eg $38 + 27 = ?$), 'find totals' or 'put answers' (McNeil et al., 2006). While an 'operational' view may suffice when solving typical elementary school arithmetic problems, it can become problematic when students encounter more complex equations in later grades (e.g.,$3x + 5 = 11; 2x - 3 = 4x + 5$). Thus, these factors may contribute to students retaining unsophisticated interpretations of the equal sign, even until the end of secondary school (Alibali et al., 2007).

Table 2. Primary school students' performance

Question	(C)	%	(IC)	%	Analysis of Incorrect Responses
Task 1. Rigid Operational: True/False Number Sentences					
1.1	0	0%	6	100%	Answer 2 (4)
					Answer 18 (2)
1.2	0	0%	6	100%	Answer 41 (2)
					No answer (4)
1.3	2	33%	4	67%	Answer 5 (4)
1.4	2	33%	4	67%	Answer 21 (1)
					Answer 4 (2)
					No answer (1)
Task 2. Flexible Operational: Open Number Sentences					
2.1	0	0%	6	100%	Answer 35 (4)
					Answer 4 (2)
2.2	0	0%	6	100%	Answer 4 (2)
					Answer 7 (4)
2.3	0	0%	6	100%	Answer 1 (5)
2.4	0	0%	6	100%	No answer (1)
					No answer
Task 3. Relational with Computational Support: Open Number Sentences					
3.1	0	0%	6	100%	Answer 4 (2)
					Answer 26 (4)
3.2	0	0%	6	100%	No answer
3.3	0	0%	6	100%	Answer 3 (2)
					No answer (4)
3.4	0	0%	6	100%	No answer
Task 4. Relational without Need to Compute: True/False Number Sentences					
4.1	0	0%	6	100%	No, because the answer not 15+11
4.2	0	0%	6	100%	No, because the result should be 9 instead of 7 + 2
4.3	0	0%	6	100%	Didn't know without computing
4.4	0	0%	6	100%	Didn't know without computing
4.5	0	0%	6	100%	No, because the results weren't 121-9
4.6	0	0%	6	100%	Didn't know what to do
4.7	0	0%	6	100%	Didn't know what to do
4.8	0	0%	6	100%	Didn't know what to do

Note: (C) = Correct, (IC) = Incorrect

Middle school students who are already proficient in arithmetic also don't seem to have a sophisticated interpretation of the equal sign. Their performance decreases as the difficulty level of the questions increases. The performance of high school students per question is presented in Table 3.

In task 1, there are four students who have difficulty in question 1.2. Those who answered 6 voluntarily reformulated the syntax such as reading 19 = 6 + 25 as '6 plus 25 equals 19' (Respondent 2, May 2019: personal interview). Apparently, there are still many high school children who only accept the equality statement of the syntactic form of expression = number (where the expression is a string operator-number) to have an operator conception of the equal sign (Jones & Pratt, 2007). Question 2.2 is the question that has the worst performance on task 2. Four respondents who answered 14 they saw an equal sign with a kind of command to complete the calculation '4 + 3 + 7' and made the answer in the box without paying attention to the operation sign and the equal sign in the sentence given math. Meanwhile, the other three respondents add $4 \times 3 = 12$ then add 7 and the result is 19. They consider that 'equivalent' is something to show that it is necessary to add answers (Darr, 2003).

Table 3. Scondary school students' performance

Question	(C)	%	(IC)	%	Analysis of Incorrect Responses
Task 1. Rigid Operational: True/False Number Sentences					
1.1	9	82%	2	8%	Answer 2 (1)
					Answer 18 (1)
1.2	7	64%	4	36%	Answer 6 (4)
1.3	10	91%	1	9%	Answer 5 (1)
1.4	9	82%	2	8%	Answer 21 (2)
Task 2. Flexible Operational: Open Number Sentences					
2.1	5	45%	6	55%	Answer 35 (4)
					Answer 15 (2)
2.2	4	36%	7	64%	Answer 14 (4)
					Answer 19 (3)
2.3	7	64%	4	36%	Answer 1 (2)
					Answer 16 (1)
					Answer 19 (1)
2.4	6	55%	5	45%	Answer 38 (2)
					Answer 24 (2)
					Answer 6 (1)
Task 3. Relational with Computational Support: Open Number Sentences					
3.1	7	64%	4	36%	Answer 26 (3)
					Answer 20 (1)
3.2	5	45%	6	55%	Answer 7 (6)
3.3	5	45%	6	55%	Answer 3 (2)
					Answer 29 (2)
					Answer 38 (1)
					No answer (1)
3.4	7	64%	4	36%	Answer 12 (3)
					Answer 3 (1)
Task 4. Relational without Need to Compute: True/False Number Sentences					
4.1	10	91%	1	9%	No, because they were not same
4.2	6	55%	5	45%	Didn't know without computing
4.3	5	45%	6	55%	No, because they were not same
4.4	0	36%	11	100%	No, because 84 was greater than 42
4.5	6	55%	5	43%	No, because they were not same
4.6	6	55%	5	43%	No, didn't know the value in the box
4.7	7	64%	4	36%	No, didn't know the value of n
4.8	7	64%	4	36%	No, didn't know the value in the box

Note: (C) = Correct, (IC) = Incorrect

Task 3 involves more than two operations in one sentence which is a special problem that can only be solved if students have a broad understanding of the equal sign (Molina & Ambrose, 2006). There are still many high school students who experience parsing obstacles as reported by (Tall & Thomas, 1991; Gunnarsson, Sonnerhed, & Hernell, 2016) as the cause of these errors. Students performed fairly well in all questions of Task 4 except in questions 4.3 and 4.4. Question 4.3 students generally say "wrong" because they see the mathematical sentence has different forms on the left and right sides of the equal sign where they believe that $8 + (3 \times 8)$ will definitely not have the same result as $(5 \times 8) - 8$. In question 4.4 all students believe that the quotient of the left and right sides of the equal sign will not be the same because it is clear that the right side has a larger number than the left side so surely the right side will have a larger quotient than the left side of the equal sign.

Table 4. College students' performance

Question	(C)	%	(IC)	%	Analysis of Incorrect Responses
Task 1. Rigid Operational: True/False Number Sentences					
1.1	12	92%	1	8%	Answer 2
1.2	11	85%	2	15%	Answer 6 (1)
					No answer (1)
1.3	12	92%	1	8%	Answer 5 (1)
1.4	12	92%	1	8%	No answer (1)
Task 2. Flexible Operational: Open Number Sentences					
2.1	11	85%	2	15%	Answer 5 (2)
2.2	12	92%	1	8%	Answer 4 (1)
2.3	12	92%	1	8%	Answer 12 (1)
2.4	10	91%	3	9%	Answer 13 (1)
					No answer (2)
Task 3. Relational with Computational Support: Open Number Sentences					
3.1	12	92%	1	8%	Answer 15 (1)
3.2	12	92%	1	8%	Answer 7 (1)
3.3	11	85%	2	15%	Answer 5 (2)
3.4	9	69%	4	31%	No answer (4)
Task 4. Relational without Need to Compute: True/False Number Sentences					
4.1	10	77%	3	23%	Didn't know without computing
4.2	11	85%	2	15%	Operations and numbers in brackets were different
4.3	11	85%	2	15%	
4.4	9	69%	4	31%	Operations and numbers in brackets were different
4.5	11	85%	2	15%	
4.6	10	77%	3	23%	The results must be different
4.7	11	85%	2	15%	No, because 76 + $45 - 9$ = 112 while $121 - 9$ =113
4.8	12	92%	1	8%	
					No, because the two boxes were different
					No, didn't know the value of n
					No, because the value of the box should be 8

Note: (C) = Correct, (IC) = Incorrect

Table 4 shows student performance. Most of the students showed good performance in every task except for questions 3.4 and 4.4. A common mistake is that they don't know multiplication by heart and aren't used to working with the triangle symbol. When the researcher asked "if I replace the triangle symbol with squares or dots, can it reduce your difficulty?" one of the respondents answered "yes". The researcher then added the question "what does it mean?" and answered "what are the results" (Respondent 3, May 2019: personal interview). This means that there are still many students and even adults who have a narrow view of symbols in mathematical sentences so that it can be one of the stumbling blocks in their performance. In question 4.4, they also have the same belief as high school students that 'the quotient of the left and right sides of the equal sign will definitely not be the same because obviously the right side has a larger number than the left side'. Although all respondents at this level have understood that the equal sign means 'the same as', they are still very dependent on the calculation process in drawing conclusions. They have not been able to draw conclusions using numerical relationships.

Student Strategies in Completing Tasks

Task 1

Task 1 involved students with four standard equations in an arithmetic context to assess students' knowledge of the equal sign. Task 1 contains a question with a rigid operational difficulty level i.e., it only succeeds on an equation in the standard format 'a + b = c' and an equal sign thinking operationally, that means 'get the answer' (Matthews et al., 2010; Rittle-Johnson et al., 2010). This item is important because it tests whether the respondent understands that the variable represents a specific and constant number value (Matthews et al., 2010). Although task 1 is at the easiest level of difficulty, there are about 38% of respondents who have difficulty answering correctly in each question item. All respondents use computational strategies in answering questions. They see an equal sign with some kind of command to complete the calculation and make an answer. So that many students, even adults, end up being 'trapped' in unreasonable calculations. When explored in the interview session, they only focused on "doing something to get results" without paying attention to the equal sign as "relation" and the operation used in the equation.

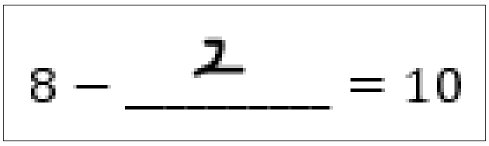

Figure 1. Examples of student misconceptions in question 1.1

The following is a snippet of an individual interview with a respondent.

Interviewer : can you explain why in question 1.1 your answer is 2?
R 4 : mmm ... because to get 10, 8 is added with 2.
Interviewer : but the operation is subtracted?
R 4 : Oh yeah ... it should be 8 subtracted 18.
Interviewer : why 18?
R 4 : because 18 subtracted 8 the result is 10.
Interviewer : Are 18 subtracted to get 10?
R 4 : yes...

This shows evidence that the concept of the equal sign gets less attention in the teaching-learning process in the early grades. This kind of situation is an obstacle that prevents individuals from internalizing the properties and meanings of arithmetic operations, from establishing relationships and even from generating deep mathematical thinking (Kiziltoprak & Kose, 2017).

Task 2

The questions in Task 2 have a flexible operating difficulty level where students are asked to solve equations in several non-standard formats. This ability is a fundamental arithmetic skill that allows writing number sentences with mathematical symbols, understanding the basic features of operations and conceptualizing numbers in various forms (Kiziltoprak & Kose, 2017). The respondents involved in this study admitted that they almost always see operations in traditional arithmetic practices, namely on the left of the equal sign and 'answer' on the right (McNeil et al., 2006).

This traditional problem format promotes an overly narrow 'left-to-right' display of equations and an equal sign. However, nontraditional problem formats are more likely to activate relational understanding of the same sign (McNeil, 2015). From Table 1 it is known that the respondents did not perform better in task 2 as compared to task 1. This situation can be considered as a fact that students learn result-oriented arithmetic and that they focus on calculations rather than the relationship between numbers and operations.

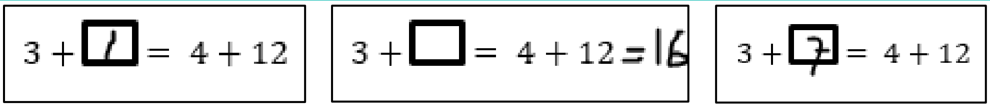

Figure 2. Examples of student misconceptions in question 2.3

Task 3

The questions in task 3 are designed to investigate students' ability to solve equations with operations on both sides. Through computational support, it is hoped that it can bring up a relational view with an operational view. These problems are designed to test students' knowledge of arithmetic equivalence properties, such as the distributive property, which have been cited as the thinking underlying formal transformation algebra (Matthews et al., 2010). The square and triangle symbols are provided to ensure students do not have to rely on certain symbols to indicate the unknown. This is expected to be a good introduction to students' understanding of variables. According to the nomological network defined by McNeil and Alibali (2005), equation solving, equation encoding, and defining the equal sign are three distinct, but theoretically related, constructs involved in children's understanding of mathematical equivalence (McNeil et al., 2011).

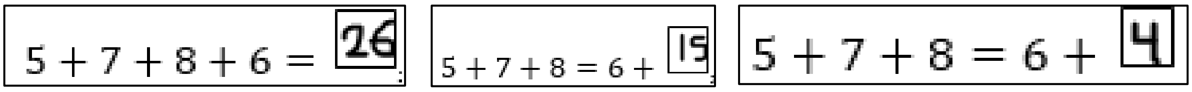

Figure 3. Examples of student misconceptions in question 3.1

For question 3.1, namely $5 + 7 + 8 = 6 + \square$, solution 26 shows the 'add all' strategy, solution 20 shows the 'what is the result' strategy, 15 shows the inaccuracy of the computational strategy carried out, and solution 4 shows an arbitrary strategy. There are some common mistakes including turning the problem into a traditional addition problem e.g. reconstructing $5 + 7 + 8 = 6 + \square$ as $5 + 7 + 8 + 6 = \square$, omitting operations on the right side of the equal sign e.g. reconstructing $5 + 7 + 8 = 6 + \square$, as $5 + 7 + 8 = \square$, and does not pay attention to the presence of the equal sign or the operations on the right and left of the equal sign.

Task 4

Task 4 provides eight statements that vary from arithmetic to algebra and in the form of true/false number sentences. Statements are given in the form of true and false number sentences that can be used to help develop a conception of the equal sign (Darr, 2003; Kindrat & Osana, 2018). Respondents were asked to indicate whether the sentence was true or false by circling the symbols '(√) or (T)' for the statement they thought was true and '(×) or (F)' for the statement they thought was wrong on the question paper. The respondents were then asked to provide reasons for the answers given as well as written justifications for their responses in the blank space provided in the questions.

All respondents, except for elementary school students, use relational-computational arguments, namely using calculations to justify their responses. They understand that the equal sign represents an equivalence relationship between two sides of the equation and confirm this equality by calculation (Kindrat & Osana, 2018; Stephens et al., 2013). This idea was confirmed at the time of the interview. They still have a very high dependence on computing to answer every question.

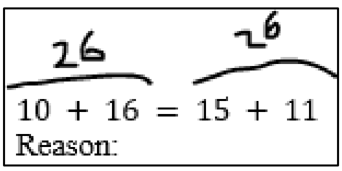

Figure 4. Examples of student response in question 4.1

Even adults who are very proficient in algebra give a 'correct' response to each question with the argument 'equal sign guarantees that the value of the left and right sides must have the same value' but the strategy used is still computational to ensure that the left side of the sign is equal to has the same value as the right side. In fact, they are expected to be able to provide arguments for numerical relationships, for example in a mathematical sentence 10 + 16 = 15 + 11 is true 'because 11 is 1 value greater than 10 as well as 16 and 15'.

Interviewer : "why do you still need to compute?"
Responden 5 : "because the left side can be not same with the right side"
Interviewer : "so, you can't give answer without any compute?"
Responden 5 : "yes"

This shows a strange contradiction, on the one hand they claim that the equal sign guarantees that the values of the left and right sides must have the same value, but still have doubts that it can be not the same. They define the meaning of 'equal value' of what they say is 'equal computational result'. Even adults are still not 'free' from the context of the equal sign as a computational result, so it is very natural that lower grade students also have this meaning. This means that all respondents have not been able to use full relational thinking, namely solving number sentences by focusing on the relationship between the numbers in the equation, instead of doing all the calculations.

Learning Experience

Students' understanding of the same as depending on the experience they gained at the beginning of elementary school arithmetic learning. The ability to interpret mathematical sentences and equations containing an equal sign relationally as a statement of mathematical equivalence is very important for students to master because it can support their ability to understand algebra (Carpenter et al., 2003; Knuth et al., 2006; McNeil, 2008). However, the results show that primary school students have great difficulty interpreting the equal sign as a relational symbol of mathematical equivalence which is referred to as the 'mathematical equivalence problem' (Kieran, 1981; McNeil, 2008; Perry, 1991). The difficulties found in this study are still experienced by some adults. The reason for these difficulties is that students build on to narrow an initial experience of arithmetic in schools which are usually taught in a very procedural way with little or no reference to the concept of mathematical equivalence. Such early experiences can lead to dependence and contribute to difficulty with high-level problems. Case in point the problem "3x=20 what is the value of x?". One of the adults who was willing to be a respondent in this study answered 17 as a solution. When asked "why 17?", the respondent answered "because 3 plus what is the result 20" (Respondent 6, May 2019: personal interview). This is one of the important findings in this study, namely that students' misconceptions about the equal sign since the early school years can become long-term misunderstandings that may even persist until higher education. These findings are consistent with previous research reports (eg, Baiduri, 2015; Knuth et al., 2006).

There are three applicable approaches to introducing a more sophisticated equal sign to children so as to provide a more meaningful arithmetic learning experience according to McNeil et al. (2015), namely: 1) focusing on explicit conceptual instructions; 2) focusing on practice with basic arithmetic facts; and 3) focusing on the role that knowledge plays in children's difficulties. The first approach aims to help children examine and reflect on the relationship between the nature of operations and numbers and express them as generalizations. The second approach aims to improve proficiency with basic facts, higher order thinking and problem solving. While the third approach aims to build a better understanding of mathematical equivalence through non-traditional problem formats, activating students' relational thinking and understanding of equivalence through relational words such as "is the same number as" instead of using the symbol "=" which is used to describe equivalence. representing equivalence, as well as setting up arithmetic problems in practice sets that allow students to induce equivalent adjunct pairs based on transitive relationships (eg if 3 + 4 = 7 and 5 + 2 = 7, then 3 + 4 = 5 + 2).

CONCLUSIONS

Based on the findings obtained in this study indicate that many students at all grade levels have not developed an adequate understanding of the meaning of the same sign. Students' understanding of the equal sign proved is not a kind of problem that is really trivial. Students' adequate understanding of the equal sign does not happen instantly because the equal sign has been introduced to students since they were in elementary school when they studied mathematics in school and they have less time to learn this symbol in the next grade (Knuth et al., 2006). Students' conceptions of equality develop in line with their previous understanding (McNeil, 2007). This misunderstanding may persist into higher education (Carpenter et al., 2003; Knuth et al., 2006). The findings of this study support this claim.

Previously, there were no studies that used the qualitative comparative analysis methods to understanding the equal sign. This study has analyzed and explored various qualitative comparative aspects in understanding the equal sign in three parts: performance, strategies, and learning experiences used in completing written assignments. So that basic things related to understanding the concept of the equal sign are obtained that are close to the real mathematical meaning and support algebraic thinking. The understanding of the concepts that students get is strongly influenced by or in line with previous understandings. Most students, both children and adults, still have operational conceptions rather than relational conceptions in understanding the same sign. They have not been able to show an understanding of the equal sign outside the context of the results of the calculation. Therefore, learning arithmetic in lower grades must instill a sophisticated understanding of the equal sign so as to support algebraic thinking skills from the start. Sophisticated understanding of the equal sign is critical to success in mathematics. For example, from 9 + 5 = 14; 9 + 5 - 3 = 14 - 3; 9 + 5 - 3 = 14 + 3; and so on, then helping students move beyond computation as a means of determining them, that is, helping them pay attention to the symmetry of the equation. As students' progress through secondary school, their opportunities to further develop a relational understanding of the equal sign should become more algebraic, for example solving equivalence equation problems (Alibali et al., 2007).

This study has limitations including the number of respondents who are small in one of the big cities in Indonesia. However, the respondents involved came from various levels of school. So that the results can be used as an illustration of students' understanding of the equal sign. It is hoped that the results of this study can provide some information to mathematics curriculum developers, mathematics book authors and mathematics teachers so that they can provide more opportunities for elementary, middle and university students to develop the concept of the equal sign correctly. Instead of waiting to introduce concepts during the middle school years, teachers should help students in elementary school to recognize the equal sign as a symptom that represents equality and balance.

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