2024, 5(3), 163–173

Journal of Didactic Mathematics
https://doi.org/10.34007/jdm.v5i3.1982

Students' conceptual understanding in analytic geometry of planes based on their prior mathematical ability

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Abstract.
Mathematical conceptual understanding is a fundamental skill that serves as a foundation for more advanced abilities, including reasoning, communication, connections, and problem-solving. However, this foundational skill is influenced by students' prior mathematical abilities. Therefore, the objective of this study is to identify and describe differences in students' mathematical conceptual understanding based on their prior mathematical abilities. This research employs a mixed-method approach, involving 87 purposively selected students enrolled in the Analytic Geometry in Planes and Space course. The research instruments include a mathematical conceptual understanding test and documentation of students' prior mathematical abilities. Data were analyzed quantitatively using one-way ANOVA and qualitatively by exploring mathematical conceptual understanding across categories of prior mathematical abilities. The findings reveal that both quantitative and qualitative analyses indicate that students with high prior mathematical abilities demonstrate superior mathematical conceptual understanding compared to those with medium or low prior mathematical abilities.

Keywords:
Mathematical concept understanding ability; prior mathematical ability; analytical geometry of the plane

How to cite:
Negara, H. R. P., Santosa, F. H., Fatimah, A. E., & Nasrullah, A. (2024). Students' conceptual understanding in analytic geometry of planes based on their prior mathematical ability. *Journal of Didactic Mathematics*, *5*(3), 163–173. https://doi.org/10.34007/jdm.v5i3.1982

INTRODUCTION

A comprehensive understanding of mathematical concepts is paramount for advancing in mathematics-related disciplines. As elucidated in various studies (Jannah et al., 2019; Pratiwi et al., 2022; Rahmadani et al., 2022), the Ministry of Education and Culture Regulation No. 58 (Antika et al., 2019; Pratiwi et al., 2022) explicitly outlines one of the objectives of mathematics education as the development of students' ability to comprehend mathematical concepts. Mardiah et al. (2020) underscore the significance of conceptual comprehension as a foundational skill that must be nurtured throughout the learning process. Furthermore, students' ability to grasp novel mathematical concepts and principles is directly correlated with their comprehension of prior concepts (Giawa et al., 2022). Consequently, individuals with a robust conceptual understanding of mathematical principles possess enhanced capabilities to effectively address a diverse range of mathematical challenges. Empirical evidence (Jannah et al., 2019; Mardiah et al., 2020; Pratiwi et al., 2022; Rahmadani et al., 2022) demonstrates that a solid grasp of mathematical concepts and principles has a positive impact on fundamental skills such as reasoning, communication, connections, and problem-solving.

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Received: November 20, 2023; Revised: March 31, 2024; Accepted: December 18, 2024
2721-5601/ © 2024 The Author(s). Published by Mahesa Research Center

Research conducted on students' comprehension abilities suggests that there is still a need for improvement in their understanding of various courses, particularly in geometry-related courses (Karim & Nurrahmah, 2018; Nasrullah et al., 2023; Dewimarni et al., 2023). Furthermore, a preliminary study conducted by researchers at an Islamic State University in Mataram City revealed that the majority of students exhibit suboptimal conceptual understanding in several courses, including Analytic Geometry in Planes and Space.

The scope of the Analytic Geometry in Planes and Space course encompasses the study of concepts pertaining to points and lines in planes and space (Halidin, 2021; Rizki, 2018; Yunita & Hamdunah, 2017). This course equips students with a comprehensive understanding of geometric concepts in planes and space (Sadidah & Sudihartinih, 2023). Typically, the concepts covered in this course are closely related to those previously acquired by students in earlier courses or during their high school education, such as circles, ellipses, parabolas, hyperbolas, and others. This connection underscores the significance of students' prior mathematical knowledge as a foundation for future learning.

The development of novel mathematical concepts is contingent upon and relies upon students' existing foundational mathematical abilities (Hasibuan et al., 2022; Mardiah et al., 2020; Pratiwi et al., 2022; Rahmadani et al., 2022). The paramountcy of prior mathematical knowledge aligns with the second dimension of Ausubel's learning theory, which elucidates how students integrate novel information with their pre-existing cognitive structures (Aprillia & Sutiarso, 2023). This aspect of the theory emphasizes how students apply novel information to their pre-existing knowledge. Students previously acquired and retained facts, concepts, and generalizations constitute cognitive structures that shape their comprehension. Meaningful learning transpires when students establish connections or associations between newly acquired information and their existing knowledge.

Prior mathematical abilities significantly facilitate deeper and more profound comprehension of mathematical concepts. When individuals can establish strong connections between novel information and their existing knowledge, the learning process becomes meaningful (Hafidzhoh et al., 2023; Purnawanto, 2022). Consequently, students' prior mathematical knowledge serves as a pivotal determinant of their success in acquiring and mastering advanced mathematical concepts. Numerous studies (Hasibuan et al., 2022; Hasibuan et al., 2023; Takaria et al., 2022) support this notion, underscoring the indispensable role of prior mathematical abilities in fostering subsequent mathematics learning.

Based on the discussion above, the researchers are interested in exploring how mathematical conceptual understanding and prior mathematical abilities can support each other. Previous studies have primarily examined mathematical conceptual understanding either quantitatively (Rendrayana et al., 2020; Sari et al., 2022) or qualitatively (Giawa et al., 2022). Similarly, studies addressing both aspects have focused on either quantitative (Antika et al., 2019; Mardiah et al., 2020; Rahmadani et al., 2022) or qualitative approaches (Pratiwi et al., 2022). This study, however, observes mathematical conceptual understanding and prior mathematical abilities through a mixed-methods approach, incorporating both quantitative and qualitative perspectives. The research questions formulated are as follows: (1) Is there a difference in students' mathematical conceptual understanding based on their prior mathematical abilities? And (2) How does mathematical conceptual understanding vary among students with high, medium, and low levels of prior mathematical abilities?

METHOD

This study adopts a pragmatic paradigm, acknowledging that data can be analyzed from both quantitative and qualitative perspectives (Creswell, 2014). Consequently, the research employs a mixed-methods approach using an explanatory sequential design. This approach commences with the collection and analysis of quantitative data to measure students' mathematical conceptual understanding (MCU), followed by qualitative data analysis to further elucidate the quantitative findings. This methodology seeks to gain a comprehensive understanding of the relationship between students' prior mathematical abilities (PMA) and their mathematical conceptual understanding.

This study was conducted at an Islamic State University in Mataram City, involving 87 students enrolled in the Analytic Geometry in Planes and Space course. The research instruments employed a combination of quantitative and qualitative instruments, as well as documentation. The quantitative instrument consisted of essay tests designed to assess students' mathematical conceptual understanding (MCU). The development of the test items was guided by MCU indicators, which encompass the following: (1) The ability to restate mathematical concepts or problems (comprehension of mathematical concepts); (2) The ability to apply concepts or computational algorithms (comprehension of mathematical operations); (3) The ability to represent mathematical situations in diverse ways and comprehend their various purposes; and (4) The ability to connect mathematical concepts internally or externally (comprehension of mathematical relations) (Kilpatrick et al., 2001). The instrument comprised four validated and reliable test items. Table 1 presents an example of the test items utilized to measure students' mathematical conceptual understanding.

Table 1. One type of question included in the MCU test instrument

Indicator	Question
The capacity to rephrase mathematical concepts or problems (comprehension of mathematical concepts)	Determine the Hess Normal Equation for the plane that passes through the points $A(3,2,1)$, $B(1,1,-2)$, and $C(2,-1,3)$.

The qualitative instrument encompasses the qualitative data obtained through an in-depth analysis of students' responses to the MCU test, supplemented by interviews to elucidate students' strategies and comprehension of the problems. Interviews were conducted with multiple students selected based on the categories derived from the quantitative test results to gain further insights into their conceptual understanding. Documentation, in contrast, pertains to information pertaining to students' prior mathematical abilities (PMA), which was extracted from the student grade records maintained by the course instructors. Subsequently, students' prior mathematical abilities were categorized into three distinct levels: high, medium, and low, as presented in Table 2.

Table 2. Number of students based on PMA

PMA Categories	Number of Students
High	8
Medium	21
Low	58

Quantitative data were analyzed using One-way ANOVA to assess variations in students' mathematical concept understanding abilities. Prior to conducting the ANOVA analysis, prerequisite tests were conducted to ensure normality and homogeneity. If the ANOVA revealed a significant difference, a post-hoc Tukey HSD was employed to identify pairs of categories that exhibited significant disparities. Qualitative data were obtained through the process of data reduction, presentation, and conclusion drawing/verification (Miles and Huberman, 1994). Data reduction commenced by selecting student responses from each PMA category (High (H), Medium (M), Low (L)). This data was subsequently summarized to highlight pertinent aspects, such as patterns of mathematical concept understanding, problem-solving strategies, and types of errors made. Interview results were also summarized to identify predominant themes or findings. The presentation of summarized data is achieved through the utilization of tables, diagrams, or descriptive narratives to facilitate interpretation. This presentation elucidates the disparities in the quality of mathematical concept understanding among PMA categories, including the representation of student responses and their respective strategies. Conclusion Drawing and Verification aims to validate or substantiate the quantitative findings. Researchers draw conclusions regarding the impact of PMA on mathematical concept understanding, ensuring that these conclusions are substantiated with robust evidence from the data. Quantitative results provide an overview of the variations in MCU among PMA categories, while qualitative results strengthen these findings by providing a detailed account of the disparities in the quality of students' responses across each category. The integration of these two analyses addresses the research objective by quantitatively identifying the impact of PMA on students' mathematical concept understanding while qualitatively elucidating the differences in the quality of understanding.

RESULTS AND DISCUSSION

The primary objective of this research study is to elucidate and characterize the disparities in students' mathematical conceptual comprehension, contingent upon their prior mathematical abilities. The research data are subjected to both quantitative and qualitative analysis (mixed-methods) to provide a comprehensive elucidation of students' mathematical conceptual understanding. The findings of the descriptive analysis are presented in Table 3.

Table 3. Analisis deskriptif MCU mahasiswa

	Categories	Mean	Std. Deviation	N
MCU	High	86.8750	8.42509	8
	Medium	59.4286	6.32117	21
	Low	33.4138	7.06867	58

Based on the descriptive data presented in Table 3, students' mathematical conceptual understanding (MCU) exhibits notable variations across the three categories of prior mathematical ability (PMA): high, medium, and low. The high PMA category demonstrates the highest mean MCU score of 86.88, accompanied by a standard deviation of 8.43. This indicates consistently high conceptual understanding among students in this category, with relatively low variation. In contrast, the medium PMA category exhibits a mean MCU score of 59.43, accompanied by a standard deviation of 6.32. This suggests a moderate level of conceptual understanding among students in this group, characterized by relatively homogeneous performance. Conversely, the low PMA category exhibits the lowest mean MCU score of 33.41, accompanied by a standard deviation of 7.07. This indicates inadequate conceptual understanding among students in this category. Notably, the slightly higher variation within this group compared to the medium PMA category suggests a disparity in understanding among students with low prior mathematical abilities.

The data clearly demonstrate a significant correlation between PMA and MCU. Students with higher PMA levels exhibit substantially superior MCU scores compared to those with moderate or low PMA levels. To further substantiate this conclusion, a one-way ANOVA test was conducted to ascertain whether the observed differences among the three PMA categories are statistically significant. Prior to hypothesis testing, prerequisite analyses, including normality and homogeneity tests, were performed. The results of these analyses are presented in Table 4 and Table 5. The analyses reveal that the mathematical conceptual understanding data for each PMA category are normally distributed ($p > 0.05$), and the data across the three PMA categories are homogeneous ($p = 0.530 > 0.05$).

Table 4. Analysis of the normality test results for data in the MCU

	Kolmogorov-Smirnov			
	PMA	Statistic	df	Sig.
MCU	High	0.108	8	0.200
	Medium	0.144	21	0.200
	Low	0.168	58	0.091

Table 5. Analysis of MCU data homogeneity test

		Levene Statistic	df1	df2	Sig.
MCU	Base on Mean	0.640	2	84	0.530
	Base on Median	0.597	2	84	0.553
	Base on Median and with adjusted df	0.597	2	82,840	0.553
	Base on trimmed mean	0.630	2	84	0.535

Following the completion of the prerequisite tests, a one-way ANOVA analysis was performed, and the results are presented in Table 6. The findings of the one-way ANOVA reveal a substantial disparity in students' mathematical conceptual understanding (MCU) in relation to their prior mathematical proficiency (PMA). The calculated F-value is 265.258, with a significance level of $p < 0.001$. $p = 0.000$. The mean MCU scores significantly differ across the PMA categories (high, medium, and low), falling below the 0.05 significance threshold. This finding suggests that PMA has a substantial impact on students' MCU scores.

Table 6. Analysis of the MCU based on the PMA

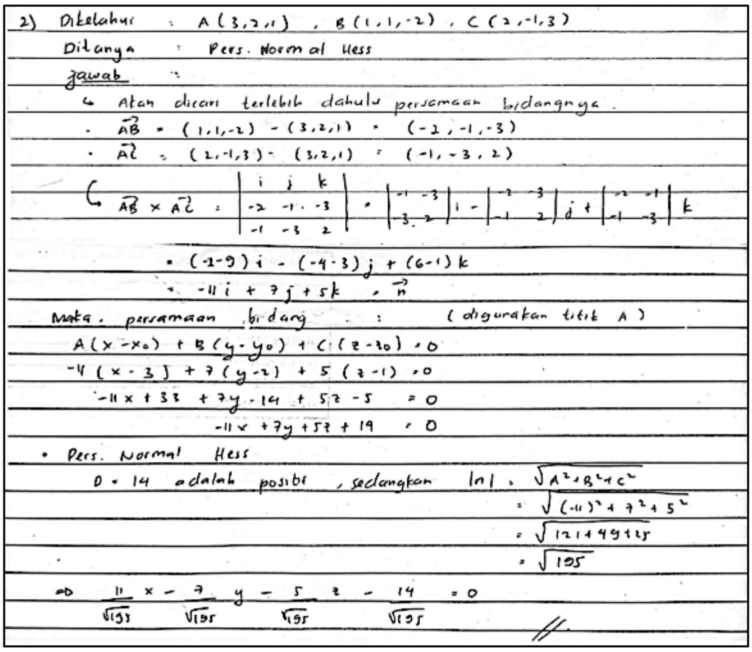
		Sum of Squares	df	Mean Square	F	Sig.
MCU	Between Groups	26172.626	2	13086.313	265.258	0.000
	Within Groups	4144.087	84	49.334		
	Total	30316.713	86			

To ascertain which category exhibited superior mathematical conceptual understanding (MCU) among the three groups, a post-hoc Tukey HSD test was performed, as presented in Table 7. The outcomes of the Tukey HSD post-hoc test provide a comprehensive analysis of the mean differences in MCU across the PMA categories. The mean difference between the high and medium PMA categories is 53.46 (p = A statistically significant difference (p-value < 0.000) was observed between the high and low PMA categories. The mean difference between these categories is also statistically significant. 27.45 (p = The mean difference is 0.000, which is also statistically significant. For the medium and low PMA categories, the mean difference is 26.01 (p = post-hoc tests revealed a significant difference (p-value < 0.000) between the mean MCU values of each PMA category and the others.

Table 7. Tukey Post-Hoc test for MCU comparisons across PMA categories

(I) PMA	(J) PMA	Mean Difference (I-J)	Std. Error	Sig.
High	Medium	53.46121	2.64904	0.000
	Low	27.44643	2.91823	0.000
Medium	Low	26.01478	1.78881	0.000

Based on the quantitative results, it can be concluded that students with high PMA exhibit significantly superior mathematical conceptual understanding (MCU) compared to those with medium and low PMA. Additionally, students with medium PMA also demonstrate superior MCU compared to those with low PMA. These findings provide compelling evidence that enhancing PMA directly correlates with improving MCU. Consequently, educational interventions focused on enhancing students' PMA, particularly for those in the medium and low categories, can be effectively implemented as a strategy to augment overall mathematical conceptual understanding. Furthermore, to elucidate the qualitative variations in mathematical conceptual understanding across the three PMA categories, a qualitative analysis was conducted based on students' MCU responses. A representative response from each PMA category was selected for analysis. The results of students' MCU responses in the high PMA category (H1) are presented in Figure 1.



2) Diketahui : A (3,2,1) , B (1,1,-2) , C (2,-1,3)

Ditanya : Pers. Normal Hess

Jawab :

↳ Akan dicari terlebih dahulu persamaan bidangnya.

· $\vec{AB} = (1,1,-2) - (3,2,1) = (-2,-1,-3)$

· $\vec{AC} = (2,-1,3) - (3,2,1) = (-1,-3,2)$

↳ $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -2 & -1 & -3 \\ -1 & -3 & 2 \end{vmatrix} = \begin{vmatrix} -1 & -3 \\ -3 & 2 \end{vmatrix} i - \begin{vmatrix} -2 & -3 \\ -1 & 2 \end{vmatrix} j + \begin{vmatrix} -2 & -1 \\ -1 & -3 \end{vmatrix} k$

$= (-2-9)i - (-4-3)j + (6-1)k$

$= -11i + 7j + 5k = \vec{n}$

Maka, persamaan bidang : (digunakan titik A)

$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

$-11(x-3) + 7(y-2) + 5(z-1) = 0$

$-11x + 33 + 7y - 14 + 5z - 5 = 0$

$-11x + 7y + 5z + 14 = 0$

· Pers. Normal Hess

$D = 14$ adalah positif, sedangkan $|n| = \sqrt{A^2+B^2+C^2}$

$= \sqrt{(-11)^2 + 7^2 + 5^2}$

$= \sqrt{121+49+25}$

$= \sqrt{195}$

$\Rightarrow \frac{11}{\sqrt{195}}x - \frac{7}{\sqrt{195}}y - \frac{5}{\sqrt{195}}z - \frac{14}{\sqrt{195}} = 0$ //

Figure 1. Students' MCU results in the high PMA category (T1)

Students in the H1 category exhibited a profound understanding of mathematical concepts (MCU). In question number 2, H1 students demonstrated systematic problem-solving skills. They commenced by determining the plane normal based on the provided three points, followed by deriving the plane equation, and ultimately expressing the Hess normal form equation correctly. This success underscores the deep conceptual understanding and strong prior mathematical proficiency of H1 students, as evidenced by their effective application of the cross-product operation to solve the problem. To substantiate this analysis, the researchers conducted interviews to corroborate the students' responses. An excerpt from the interview transcript between the researcher and a H1 student is provided below.

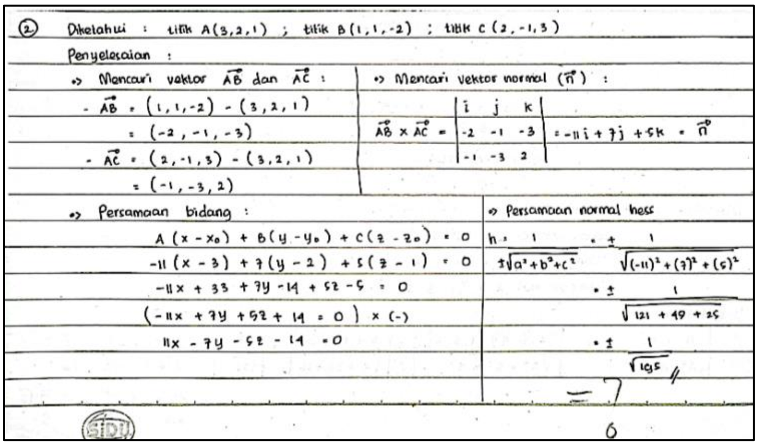
Researcher : Could you elaborate on your approach to solving question number 2? Did you employ any specific techniques or strategies?

H1 : To address question number 2, I commenced by determining the plane normal using the three provided points. Subsequently, I derived the plane equation utilizing the appropriate formula. Subsequently, I proceeded to the final step of expressing the Hess normal form equation. My strategy was to initially grasp the fundamental concept, namely the cross product, as it is pivotal to solving the initial steps.

Researcher : Did you encounter any challenges in solving this problem?

H1 : Notwithstanding. I possess a comprehensive understanding of the cross product and vector operations, necessitating only verification that my steps adhered to the theoretical framework.

Referring to the interview conducted, students in the high PMA category exhibited a comprehensive and structured comprehension of mathematical concepts. This was evident from the sequential and logical manner in which they explained the problem-solving steps. H1 successfully identified the cross product as the fundamental concept for solving question number 2. Additionally, they demonstrated a high level of confidence in their conceptual understanding, supported by a robust theoretical foundation. The interview corroborates that students in this category not only grasp the concepts but also apply them accurately. The results of the mathematical conceptual understanding for the medium PMA category (M1) are presented in Figure 2.



② Dikelahui : titik $A(3,2,1)$; titik $B(1,1,-2)$; titik $C(2,-1,3)$

Penyelesaian :

⇒ Mencari vektor $\vec{AB}$ dan $\vec{AC}$:

- $\vec{AB} = (1,1,-2) - (3,2,1)$

$= (-2,-1,-3)$

- $\vec{AC} = (2,-1,3) - (3,2,1)$

$= (-1,-3,2)$

⇒ Mencari vektor normal $(\vec{n})$:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -2 & -1 & -3 \\ -1 & -3 & 2 \end{vmatrix} = -11i + 7j + 5k = \vec{n}$$

⇒ Persamaan bidang :

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$
$$-11(x-3) + 7(y-2) + 5(z-1) = 0$$
$$-11x + 33 + 7y - 14 + 5z - 5 = 0$$
$$(-11x + 7y + 5z + 14 = 0) \times (-)$$
$$11x - 7y - 5z - 14 = 0$$

⇒ Persamaan normal hess

$$h = \frac{1}{\pm\sqrt{a^2+b^2+c^2}} = \pm\frac{1}{\sqrt{(-11)^2+(7)^2+(5)^2}}$$
$$= \pm\frac{1}{\sqrt{121+49+25}}$$
$$= \pm\frac{1}{\sqrt{195}} //$$

6

Figure 2. Students' MCU results in the medium PMA category (M1)

Students in the M1 category generally demonstrated a satisfactory comprehension of mathematical concepts, albeit not as robust as those in the H1 category. In question number 2, M1 students were able to follow the same procedures as H1 students in determining the plane normal and deriving the plane equation. However, they were unable to complete the final step, which was to express the Hess normal form equation. This suggests that while their conceptual understanding is reasonably sound, there are deficiencies in maintaining consistency in mathematical conceptual understanding during the concluding stages. To further elucidate their understanding and corroborate their observations, the researchers conducted interviews with M1 students. An excerpt from the interview transcript between the researcher and an M1 student is provided below.

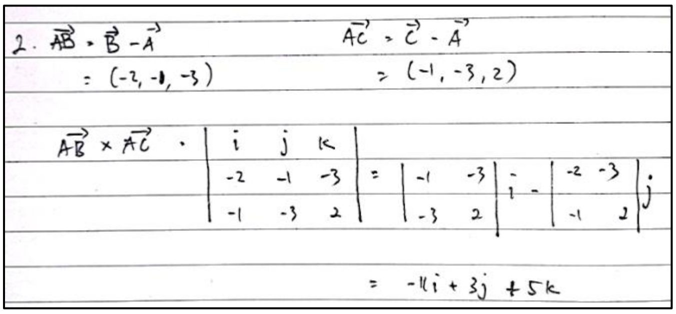
Researcher : Could you elaborate on the methodology employed to resolve question number 2?

M1 : Initially, I determined the plane normal and derived the plane equation as per the class teachings. However, I encountered difficulties in expressing the Hess normal form equation. Consequently, I am uncertain about the correctness of my result, as I lack a comprehensive understanding of the final step.

Researcher : In your opinion, what was the primary obstacle in solving this problem?

M1 : I believe that my comprehension of the relationship between the plane equation and the Hess normal form equation is lacking. While I am proficient in utilizing the cross product, I lost track of the subsequent steps.

Based on the interview results presented above, students in the medium PMA category exhibited a generally satisfactory comprehension of mathematical concepts, albeit with inconsistencies. During the interview, they acknowledged their ability to follow the initial steps in solving problems, such as determining the plane normal and deriving the plane equation. However, they encountered difficulties at the final stage, which involved expressing the Hess normal form equation. This suggests that while M1 students possess adequate operational abilities, they lack a profound understanding of the relationships between concepts, particularly those that are abstract. Consequently, it is evident that additional learning is necessary to facilitate the connection between mathematical concepts. The results pertaining to the mathematical conceptual understanding for the low PMA category (L1) are depicted in Figure 3.



2. $\vec{AB} = \vec{B} - \vec{A}$ $\qquad$ $\vec{AC} = \vec{C} - \vec{A}$

$= (-2, -1, -3)$ $\qquad$ $= (-1, -3, 2)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -2 & -1 & -3 \\ -1 & -3 & 2 \end{vmatrix} = \begin{vmatrix} -1 & -3 \\ -3 & 2 \end{vmatrix} i - \begin{vmatrix} -2 & -3 \\ -1 & 2 \end{vmatrix} j$$

$$= -11i + 3j + 5k$$

Figure 3. Students' MCU results in the low PMA category (L1)

Students in the L1 category exhibited a limited comprehension of mathematical concepts. In question number 2, L1 students were only able to complete the initial step, which was the cross-product calculation. However, there were errors in the calculation process. The provided answers did not reach the stage of determining the Hess normal form equation, suggesting a weak understanding of the fundamental concepts necessary to solve the problem. To substantiate this observation, the researcher conducted a follow-up interview with L1 students. An excerpt from the interview transcript between the researcher and an L1 student is provided below.

Researcher : Could you elaborate on your approach to solving question number 2? Were there any particular sections that posed significant challenges?
L1 : Initially, I attempted to apply the cross-product method, as it had been previously demonstrated. However, I encountered confusion thereafter, leading to incorrect calculations.
Researcher : In your opinion, what factors contributed to the difficulty in solving this problem?
L1 : I believe that a fundamental understanding of the underlying concepts, such as calculating the plane normal or determining the Hess normal form equation, is lacking. The sequential steps were challenging to follow due to my insufficient grasp of the concepts from the outset.
Researcher : Do you believe that the provided class material adequately addressed the necessary prerequisites for this problem?
L1 : While the material was presented, I have yet to fully master the fundamental concepts. I recognize the need for additional study in vector operations before I can fully comprehend this concept.

In light of the interview results presented, students categorized as low PMA exhibited a severely limited comprehension of mathematical concepts. As evidenced by the interviews, they were only capable of identifying the initial steps of the solution, such as the cross product, but their calculations were plagued by errors, preventing them from progressing to the subsequent step. Additionally, L1 students also encountered difficulties in grasping fundamental concepts, which ultimately hindered their ability to solve the problem effectively. This underscores the paramount importance of providing a comprehensive reinforcement of basic conceptual understanding prior to enabling students to acquire and apply more intricate concepts.

A comprehensive analysis of the responses and interviews reveals a robust correlation between PMA and the pattern of mathematical understanding exhibited by students in the MCU. Students with high PMA demonstrate a superior comprehension of mathematical concepts and possess the ability to solve problems systematically. Students with moderate PMA exhibit potential but necessitate intervention to address inconsistencies in their understanding. Conversely, students with low PMA require intensive foundational learning to enhance their mathematical conceptual understanding.

The qualitative data findings further corroborate the quantitative analysis, demonstrating that students with initial mathematical aptitude exhibit a superior comprehension of mathematical concepts compared to those with moderate or low aptitude. These findings align with previous research conducted by Hasibuan et al. (2022; 2023) and Takaria et al. (2022), which elucidate the significance of initial mathematical ability as a prerequisite for establishing a solid foundation for subsequent mathematical learning. Initial mathematical ability serves as a pivotal stepping stone, enabling students to attain advanced capabilities. Every learning process is rooted in the initial mathematical abilities of students, which are progressively developed into novel skills that serve as objectives within the learning trajectory. Each of these newly acquired abilities represents the culmination of the learning process (Hafidzhoh et al., 2023; Aprillia & Sutiarso, 2023; Purnawanto, 2022).

Students with robust initial mathematical abilities demonstrate enhanced material absorption compared to those with lower initial abilities when presented with challenging material accompanied by appropriate learning strategies (Aprillia & Sutiarso, 2023). This phenomenon can be attributed to the fact that students with high PMA possess the foundational mathematical skills necessary to engage with advanced concepts, given the hierarchical, structured, and continuous nature of mathematics.

The initial mathematical aptitudes possessed by students significantly influence their acquisition of new information. Differences in initial abilities to process and integrate learned material can lead to varying levels of conceptual understanding, particularly in restating concepts, classifying objects based on conceptual criteria, providing examples and non-examples of concepts, presenting concepts, and providing necessary and sufficient conditions. This can result in diverse learning outcomes for students. Concepts, usage, application, selection of specific processes or procedures, and the implementation of algorithms are all impacted. Consequently, disparities in initial mathematical abilities will affect students' ability to comprehend mathematical concepts.

The impact of students' initial mathematical abilities on their comprehension of mathematical concepts can be observed by noting that students who demonstrate strong conceptual understanding are those whose initial mathematical abilities fall within the moderate or high categories. Conversely, students with limited conceptual understanding also tend to have limited initial mathematical abilities. These findings align with those presented by Aprillia & Sutiarso (2023), which assert that students with higher initial mathematical abilities encounter fewer difficulties in comprehending the material to be learned. However, if a student's initial mathematical ability is inadequate, they will encounter challenges in grasping the material and, consequently, struggle with acquiring new concepts.

CONCLUSIONS

Based on the findings and discussion, it can be concluded that students' mathematical conceptual understanding (MCU) varies significantly based on their prior mathematical ability (PMA). Quantitatively, the analysis reveals that the average MCU scores of students with high PMA are superior to those with medium and low PMA. Qualitatively, students with high PMA exhibit more structured, fluent, and in-depth responses in expressing their mathematical concept understanding. Conversely, students with medium PMA demonstrate a relatively good understanding of concepts, but there are weaknesses in the consistency and depth of their answers. Meanwhile, students with low PMA exhibit limited conceptual understanding, with significant difficulties in solving problems and explaining fundamental concepts.

The implications of these findings for mathematics education are substantial, particularly in the design of learning that acknowledges students' initial abilities. Identifying students' relevant prior mathematical abilities to the new concepts to be learned can serve as an initial step in enhancing conceptual understanding. This approach can be implemented through diagnostic assessments at the commencement of the course to identify students' PMA levels and design appropriate learning strategies, such as providing enrichment materials for students with low PMA. Furthermore, this study emphasizes the significance of strengthening foundational concepts prior to studying more complex material to facilitate students' comprehension of the relationships between mathematical concepts.

Several limitations exist in this study. Firstly, the research was conducted at a single educational institution with a limited sample size, potentially reducing the generalizability of the findings to a broader context. Secondly, the assessment of students' initial mathematical abilities was solely based on archived academic records, which may not fully capture their actual proficiency. Thirdly, while the interviews provided valuable qualitative insights, the relatively small number of participants may limit the representativeness of the qualitative data. These limitations warrant future research involving a larger sample size, more comprehensive PMA measurement methods, and triangulation of qualitative data through additional in-depth interviews. Taking these implications and limitations into account, future research endeavors aim to expand upon these findings to support the development of more effective and inclusive mathematics education.

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