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Optimizing students' mathematical problem-solving abilities through geoboard-assisted didactic design on triangular number pattern material

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Abstract. The research aimed to optimize students' mathematical solving abilities through the implementation of a didactic design developed based on lesson study activities on triangular number patterns. The research design used was didactical design research. Participants in the study were 10 students (12–15 years old) at a junior high school in Mataram, Indonesia. The researcher was the main instrument, with several additional instruments, one of which was the didactic hypothesis design. The data were analyzed using qualitative analysis. The research results revealed an increase in students' mathematical problem-solving abilities during the implementation of didactic design on triangular number pattern material. However, the increase that occurred was less than optimal. This increase resulted from the didactic design encouraging students to use *Quizizz*, geoboards, and inspirational videos to solve problems. However, when watching inspirational videos and *Quizizz*, there were problems, such as an unstable internet connection, so activities that should have been done in groups were done collectively for one class.

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INTRODUCTION

Number patterns are one of the mathematical concepts studied in school (Risdiyanti & Prahmana, 2020). One example of the number pattern concept studied is the triangular number pattern. As one of the concepts studied at school, triangular number patterns actually become a problem for students (Wabang et al., 2023). Likewise with problem-solving abilities (Wahyuni et al., 2023). In fact, triangular number patterns have quite a role, both in other scientific disciplines and in mathematics itself (Garge & Shirali, 2012; Ulness, 2022). For example, in the engineering field, the concept of triangular number patterns is used as a tool to build triangular structures on bridge frames. Quite a lot of research has studied triangular number patterns, but not many have tried to study how to optimize mathematical problem-solving competence through lesson study activities based on didactical design research. In fact, problem-solving ability is one of the standard processes for learning mathematics.

For example, the research of Utomo et al. (2022) aimed to discover students' errors when solving number pattern problems at one junior high school in Central Bengkulu, Indonesia. This research has used a descriptive research design, and one conclusion has been obtained: the errors made by students when solving triangular number pattern problems have been 63,33%. Still with the same research design, Fatahillah et al. (2023) have studied the description of students' conceptual understanding abilities for number pattern material at a junior high school in Krain, Indonesia. The results of this research have revealed that some participants have not been able to categorize whether a number was included in the triangular number pattern or not. In contrast to the two previous studies, research conducted by (McMartin & McMaster, 2016) in Sidney, Australia, has designed learning for triangular number patterns. This research has revealed that figural understanding should be emphasized before numerical understanding when teaching triangular number patterns.

In contrast to several previous studies, this research seeks to provide an alternative solution to optimize students' mathematical problem-solving abilities in learning triangular number patterns by compiling a didactic design through lesson study activities. This activity is used because it is able to provide various points of view or suggestions for improvements to planning, implementation, and reflection on learning (Chen & Li, 2010; Fitriati et al., 2023). The didactic design that is prepared is then assisted by a geoboard to make it easier to solve problems (Ningrum & Napitupulu, 2021; Vázquez-Serrano et al., 2022) and integrates the use of ICT, such as inspirational videos from YouTube and Quzizz, so that student learning motivation increases (Coles, 2014; Khasanah & Lestari, 2021; Santagata et al., 2021; Saragih & Panjaitan, 2023; Yanuarto & Hastinasyah, 2022). The problem-solving abilities intended in this research adapt aspects of problem-solving by Polya. These four aspects include understanding the problem, formulating a plan, executing the plan, and looking back (Lertyosbordin et al., 2021). There are several research questions derived from the research objectives, including:

1. What is the initial condition of students' mathematical problem-solving abilities before implementing the hypothetical didactic design?
2. What form of didactic design is hypothesized to be able to optimize students' mathematical problem-solving abilities?
3. How does a hypothetical didactic design be implemented to optimize students' mathematical problem-solving abilities?
4. What is the form of revision of the didactic design of the hypothesis for optimizing students' mathematical problem-solving abilities?
5. What is the condition of students' mathematical problem-solving abilities after implementing the hypothetical didactic design?

METHOD

The design of this research was didactical design research (DDR). DDR was used for two reasons. First, the interpretive paradigm in DDR was quite relevant to research questions related to the factors that caused students to experience learning obstacles, especially when solving problems. Second, research questions related to the form of didactic design for learning triangular number patterns were relevant to the study using the critical paradigm found in DDR. The didactic design in this research took the form of student worksheets, which were prepared using epistemic learning patterns. The epistemic learning pattern was used because the learning activities in this learning pattern were epistemic in nature by following various didactic situations during learning (Sidik et al., 2021; Sukarma et al., 2024; Suryadi, 2019a).

The procedure in this study then followed the DDR steps, namely prospective, meta-pedadidactic, and retrospective analysis (Marfuah et al., 2022; Suryadi, 2019b). To make each DDR step effective and efficient, this research followed the activities in the lesson study. The lesson study activity was chosen because it was able to provide various points of view regarding learning from several teachers, and the lesson study steps tended to be relevant to the DDR steps. The planned activity in the lesson study was relevant to prospective analysis in DDR; the do activity was relevant to meta-pedagogical analysis; and the seen activity was relevant to retrospective analysis (Huang & Shimizu, 2016; Joubert et al., 2020). For more clarity regarding the research procedures, see Figure 1.

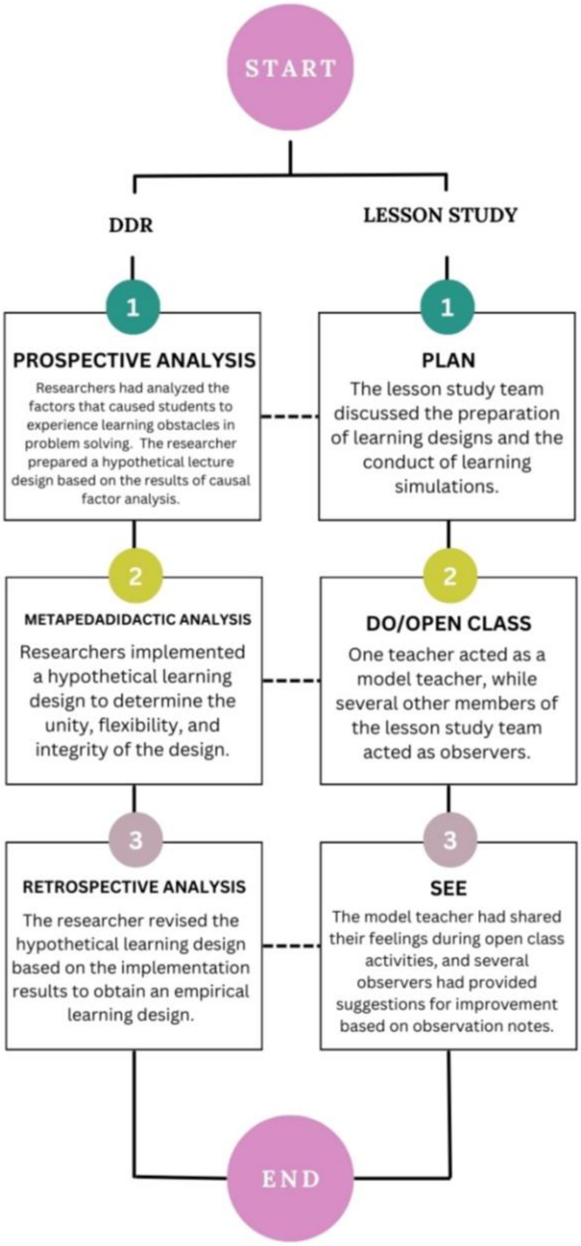

Figure 1. Research procedure

This research was conducted at a private school in Mataram, Indonesia. This school was chosen because, based on the results of a baseline survey of students at the school, it was indicated that the majority of students experienced learning obstacles. Participants in this research were 10 class VIII students. These participants were selected using purposive sampling techniques. The students were 12–15 years old; 6 were male, and four were female; the parents' profession was mostly farming with a high school educational background; 5 people were from the Sasak tribe, while 5 others were from the Samawa tribe. The researcher acted as the main instrument in this research by involving several additional instruments. These additional instruments included a didactic hypothesis design, a problem-solving ability observation sheet, a student interview guide, a student activity observation sheet, and a documentation study. The hypothetical didactic design was used to obtain data related to learning implementation. The hypothetical didactic design used in this research obtained a content validity ratio (CVR) value of 1 out of 7 experts. In other words, this design was considered essential for use in research (Lawshe, 1975). The problem-solving ability observation sheet was used to obtain data on students' problem-solving abilities during learning. Interview guides were used to obtain in-depth information regarding student performance. Student activity observation sheets were used to obtain student activity data during learning. Meanwhile, documentation studies were used to obtain authentic data related to student activities during learning.

The data in this research was analyzed using qualitative data analysis. The stages of the analysis were data reduction and presentation, as well as drawing conclusions (Miles et al., 2014). During data reduction, researchers discarded some data that was considered less important. For example, researchers did not record all activities in learning; instead, they only focused on interesting student activities. At the data presentation stage, researchers presented data that had been reduced in various forms of representation, such as event descriptions, tables, or pictures. When drawing conclusions, researchers related research results to research questions, existing theories, and previous research. This activity was intended to obtain findings or answers to research questions that had been previously created. To strengthen the trustworthiness of the research, researchers used data triangulation in the form of triangulation of sources and data collection methods (Heale & Forbes, 2013).

RESULTS AND DISCUSSION

Prospective Analysis/Plan

What is the initial condition of students' mathematical problem-solving abilities before implementing the hypothetical didactic design?

Before developing a hypothetical didactic design, the researcher had identified students' problem-solving abilities by observing their performance when solving problems. After analyzing qualitative data on the results of performance observations, information was obtained that more than 50% of students had not been able to demonstrate indicators of problem-solving ability, and 100% of students had not been able to solve problems correctly. As evidence, only 4 out of 10 participants were able to develop and implement problem-solving strategies. For more details, refer to Table 1.

Table 1. Percentage of students' mathematical problem-solving ability based on indicators

Indicator	Percentage
Identify problem.	0
Formulate problem-solving strategies.	40
Implement problem-solving strategies.	40
Checking back.	0
Correct solution.	0

The results of this research are then in line with several previous studies that reveal students' problem-solving abilities when solving mathematics problems tend to be quite low (Maksum et al., 2021; Mudhofir, 2021; Pohan et al., 2020). According to the interviews, one of the reasons students had learning obstacles and had low problem-solving skills was that math lessons at school did not help them build concepts or formulas through problem-solving. Excerpts from the interview results can be seen in Table 2. This reason is then in line with several previous studies (Nurhayati et al., 2021; Prema & Sathiskumar, 2021) that reveal one of the factors causing students to experience problems when solving problems is that mathematics teachers tend to apply conventional learning or rarely use problems as one of the situations in constructing concepts or formulas in mathematics.

Table 2. Excerpts from in-depth interviews with students

Researcher Questions	Student Answers
How is the mathematics learning process in your class?	*Just normal, sir.*
It means?	*Yes, as usual, sir. The teacher gives formulas, gives example questions, and then practices.*
Is that so?	*Yes, Sir.*
Does that mean you were never taught to find formulas through problem-solving activities?	*I don't think so, sir.*

What form of didactic design is hypothesized to be able to optimize students' mathematical problem-solving abilities?

After learning about the factors that caused students to experience learning obstacles when solving problems, researchers and a team of lecturers prepared a didactic design. The didactic design that was prepared then facilitated students by providing problems as initial situations for students to find numbers that fell into the triangular number pattern. Problems served as sufficient scaffolding when educators wanted to invite students to construct a mathematical concept or formula (Sukarma et al., 2024; Suryadi, 2019a). A snapshot of the problem integrated into the hypothetical didactic design can be seen in Figure 2.

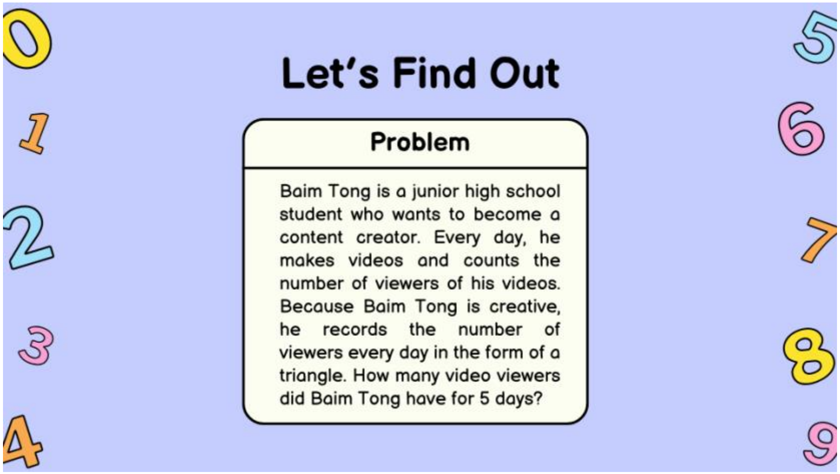

Figure 2. Problem snippets integrated into hypothetical didactical design

Furthermore, learning consisted of at least three activities, namely initial, core, and closing activities (Aylward, 2012). Therefore, the hypothetical didactic design developed in this research facilitated students to go through all these activities. This hypothetical didactic design was often also referred to as an epistemic learning pattern (Sukarma et al., 2024) because it facilitated students in constructing mathematical concepts through stages of philosophical knowledge acquisition. These activities included several activities, namely initial activities *(Let's Read, Let's Watch,* and *Let's Play)*, core activities *(Let's Find Out, Let's Tell Stories, Let's Conclude,* and *Let's Practice)*, and closing activities *(My Reflection)*.

In more detail, the *Let's Read* activity was related to students reading a description of the learning objectives in the didactic design. This activity was intended so that students understood the targets that must be achieved at the end of learning (BSKAP, 2022). *Let's Watch* contains activities to watch inspirational videos related to the benefits of number patterns in everyday life. This activity aimed to attract students' motivation or interest in learning through the videos they watched (Coles, 2014; Febliza et al., 2023; Santagata et al., 2021; Wijnker et al., 2021). *Let's Play* contained student activities for playing educational online games using Quizizz. This activity aimed to confirm students' prerequisite knowledge before entering the material to be studied through play activities with online-based test applications (Khasanah & Lestari, 2021; Yanuarto & Hastinasyah, 2022).

Let's Find Out contained student activities for solving problems as a means of discovering the concept of triangular number patterns. In this activity, students were assisted with geoboard media (Ningrum & Napitupulu, 2021; Vázquez-Serrano et al., 2022) to make it easier for students to find the numbers that formed a triangle. After this activity was finished, students continued the activity by telling all the students in the class about the solutions and processes that had been followed. Other students could provide responses or confirm the answers of students who were telling stories (Ahmad, 2021). This activity aimed to obtain validation of solutions related to the problems being solved (Arslan et al., 2011). This activity was termed *Let's Conclude*. The final goal of this activity was that students were able to find numbers that were included in the triangular number pattern or not (Sukarma et al., 2024). The next activity was *Let's Practice*. In this activity, students were expected to be able to implement previously discovered concepts to solve problems in different contexts or situations than before (Suryadi, 2019b). This activity aimed to make students' understanding of the concepts better than before. Lastly, *My Reflection* was a student activity when checking back on understanding, feelings, and suggestions for improvement for future learning (Cullinane, 2011; Ghorbanpour et al., 2021; Yang & Xin, 2022).

Furthermore, the core activities in the didactical design of the hypothesis in this research were developed using the theory of didactical situations. This theory divided learning activities into several situations, namely action-formulation, validation, and institutionalization situations (Brousseau, 2002; Prabowo et al., 2022; Sidik et al., 2021). Action-formulation situations were situations when students used mental and physical activities to solve problems according to previous knowledge and experience. The *Let's Find Out* activity represented this situation. A validation situation was a situation where students validated concepts discovered through previous problem-solving activities. The *Let's Tell Stories* and *Let's Conclude* activities represented this situation. An institutionalization situation was a situation when students used mathematical concepts that had been discovered in solving other problems that involved those concepts. The *Let's Practice* activity (Arslan et al., 2011; Sukarma et al., 2024) served as a representation of this situation.

Metapedadidactic Analysis/Do

What is the process of implementing a hypothetical didactic design to optimize students' mathematical problem-solving abilities?

After completing the hypothetical didactic design and carrying out simulations, the model teacher facilitated open-class activities. As usual, at the beginning of the learning activity, the model teacher opened the lesson with greetings and an opening prayer. After that, the model teacher asked how each student was doing while checking the student's attendance. The model teacher then asked the students to gather in their respective groups. After the students gathered, the student group was given a *Chrome Book* to use as a device to open the hypothetical didactic design.

Learning activities began with *Let's Read*. In this activity, students were asked to read the existing learning objectives. Several students then read, and the model teacher read the learning objectives again. The next activity was *Let's Watch*. The initial plan was that each group would watch using their own Chromebook. However, the model teacher finally decided to use an LCD when watching because all the student groups' Chrome Books had internet network problems. In this activity, the model teacher played the inspirational video twice because, on the first screening, there were no students who could provide comments. With several trigger questions, in the second screening, several students were able to reveal that number patterns are quite often found in everyday life, such as motifs of butterfly wings, snail shells, sunflowers, and dragonfly wings. The next activity was *Let's Play*. Each group did not carry out play activities using a Chrome Book because the internet connection had not yet improved, but the model teacher continued to use an LCD. In other words, students were asked to answer the questions on Quizizz directly and simultaneously. In this activity, students seemed capable of answering the questions. In other words, students indicated that they did not experience problems related to prerequisite knowledge.

After finishing playing, students were asked to solve the problems in the *Let's Find Out* activity. Each group of students was given a geoboard to assist in finding solutions to the problems contained in the hypothetical didactic design. In this activity, obstacles were found, such as students having difficulty understanding the problem. In response to this, the model teacher asked students to read the problem again, and it was seen that students were starting to develop strategies using the geoboard they had. When using the geoboard, problems were also found, such as the rubber bands often coming loose on the geoboard, forcing student groups to recreate the triangular shape. One example of documentation when students used a geoboard can be seen in Figure 3.

Figure 3. Students use geoboards when learning

Apart from that, when carrying out the problem-solving strategy, students did not appear to write anything on the worksheet. The model teacher also asked groups of students to write down the number of nails contained in the triangle shape. Students were then seen recording the numbers obtained. Excerpts of students' answers can be seen in Figure 4. The model teacher also asked students to check the numbers written again, but because the allocated time had run out, they continued with the *Let's Tell Stories* activity. In the *Let's Tell Stories* activity, obstacles were also found, such as groups of students tending to have difficulty determining group members who would make presentations. After a few minutes, representatives from each group presented their solutions.

Furthermore, students appeared capable of concluding that numbers such as 1, 3, 6, 10, and 15 formed a triangular number pattern. Students concluded that these numbers were a triangular number pattern because they saw that when making a triangle on a geoboard, the number of nails contained in the triangle was 1, 3, 6, 10, or 15. The model teacher also provided additional information that the general formula that could be used to confirm whether a number formed or belonged to a triangular number pattern is $U_n = \frac{1}{2}n(n+1)$, where U_n is the third term and $n = 1, 2, 3, ...$ After the activity concluded, the activity should have continued with *Let's Practice*. However, because there was not enough learning time, this activity was skipped, and students were asked to do the *My Reflection* activity. When reflecting on the concept of triangular number patterns, several students appeared to be mistaken, and several students felt sad during lecture activities.

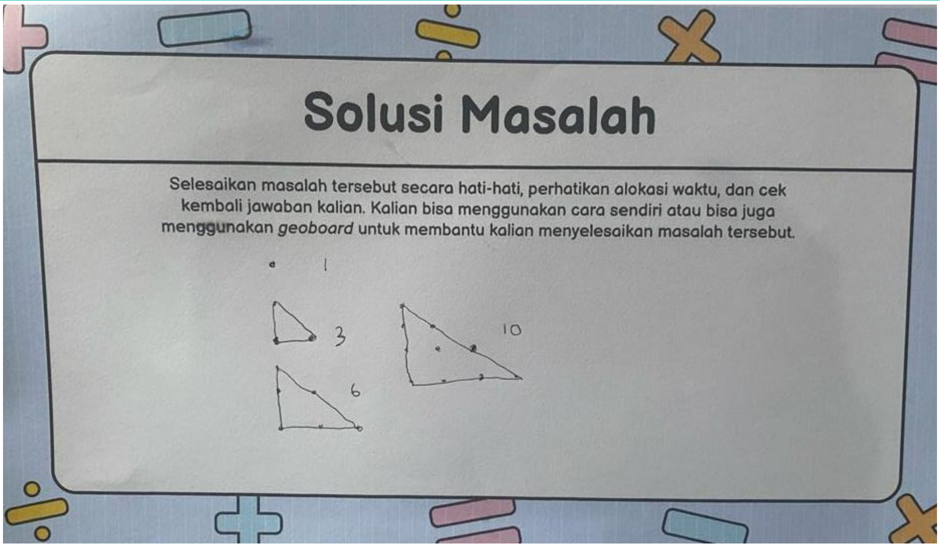

Figure 4. Excerpts of student answers in the *let's find out* activity

The results of this research are now in line with several previous studies (Azhari & Fajri, 2021; Isnawan et al., 2022; Moliner et al., 2022), which reveal that unstable internet connections tend to be one of the obstacles to ICT-assisted learning. The results of this research are also in line with several previous studies (Khalafov, 2021; Mathematical Sciences HE Curriculum Innovation Project, 2012), which reveals that learning that integrates problems tends to take a relatively long time in classroom learning. This is because the activities carried out by students for problem-based learning tend to be more numerous than conventional learning (Mudhofir, 2021; Pohan et al., 2020).

Retrospective Analysis/See

What is the form of revision of the didactic design of the hypothesis for optimizing students' mathematical problem-solving abilities?

After the implementation activities were completed, the researcher and a team of lecturers and teachers carried out the observed activities. In this activity, the model teacher conveyed feelings during learning. The model teacher felt nervous at the beginning of the learning activity. Although, in core activities and beyond, teachers felt more comfortable when teaching. The results of this research are in line with several previous studies (Norhanah, 2022; Vlorensius et al., 2016), which revealed that model teachers tend to feel nervous when carrying out open-class activities because there are quite a lot of observers observing.

Based on the results of the analysis of student activity observation sheets, information was obtained that there were several student activities that did not appear to be optimal. First, students tended to be less able to provide opinions when asked by the model teacher about the message they wanted to convey from the inspirational video. Second, students could not do *Let's Play* activities in groups because they were hampered by an unstable internet connection. Third, students did not appear to be dexterous when using geoboards. The rubber on the geoboard tends to come off when making triangles. Fourth, students appeared less than optimal when writing processes or problem solutions on worksheets. Fifth, the *Let's Practice* activity could not be carried out due to a lack of time. Sixth, most students felt sad at the end of learning activities. Following up on these obstacles, the researcher, together with a team of teachers and lecturers, prepared an alternative solution in the form of a redesign, which can be seen in Table 3. The revised hypothetical didactic design was then referred to as an empirical didactic design. This design is classified as an epistemic learning pattern (Sukarma et al., 2024) because it was able to facilitate students in constructing concepts using mental and physical actions, as well as epistemic steps when constructing mathematical concepts.

Table 3. Revision of the didactical design hypothesis

Obstacles	Revision
Inability to express opinions.	Develop trigger questions to guide students.
Let's Play activities are not carried out in groups.	Paper has taken the place of *Let's Play* activities.
Students are not used to using geoboards.	Replacing geoboard media with cardboard media.
Students are less than optimal when writing solutions on worksheets.	Ask students to divide tasks when solving problems.
Time allocation that does not cover all activities.	Ask students to divide tasks when solving problems.
Some students feel sad after learning takes place.	Look for activities that are more relevant to students, such as using cardboard or making posters.

What is the condition of students' mathematical problem-solving abilities after implementing the hypothetical didactic design?

During the study, the researcher, together with a team of teachers and lecturers, analyzed the results of the student performance observation sheet when solving problems. The results of the analysis revealed that there was an increase in the percentage of students who were able to demonstrate most indicators of mathematical problem-solving ability. However, this increase was quite low, namely a maximum of around 20%. In fact, for the indicator of checking back, there was no increase in the percentage of students. The comparison between the initial condition and the final condition of mathematical problem-solving abilities can then be seen in Figure 5.

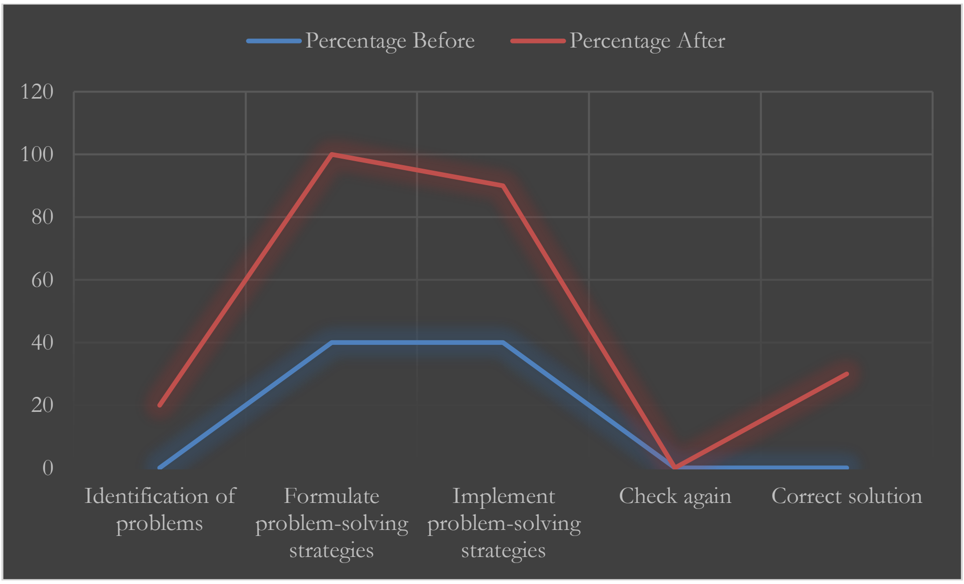

Figure 5. Comparison of the percentage of mathematical problem-solving ability indicators

As previously described, there was an increase in the percentage of students who were able to demonstrate performance for most indicators of mathematical problem-solving ability. The results of this research are then in line with several previous studies (Raza et al., 2020; Surya et al., 2017), which reveal that problem-based learning tends to optimize students' mathematical problem-solving abilities. This is because problem-based learning tends to provide students with more opportunities to explore ideas or strategies for solving problems (Cooper & Alibali, 2012; Idawati et al., 2020). The results of this research are also in line with several previous studies that reveal that learning design based on didactic situations (Isnawan, 2022; Sukarma et al., 2024) and structured through lesson study activities (Fitriati et al., 2023; Haryoto & Narimo, 2013) tends to have a positive impact on the development of student competence.

If you look again at Figure 5, you can find information that there is no increase in the percentage of students who show indicators of checking again. It is noted that no student checks their answers again, either before or after implementation. One of the reasons why students do not check back is because they do not have enough time to carry out these activities. Students seem to spend more time solving problems. The results of this research are then in line with research (Mudhofir, 2021; Pohan et al., 2020), which reveals that the time allocated when solving problems tends to be more, which has an impact on subsequent learning activities, and students tend to rarely check their answers again after solving problems.

CONCLUSIONS

Based on the previous description, it can be concluded that the geoboard-assisted hypothetical didactic design can improve students' mathematical problem-solving abilities in learning triangular number patterns. Although the increase that occurs is not optimal, this is because the learning design integrates problems as a situation for students to consider when constructing mathematical concepts. Apart from that, the design also integrates inspirational videos and Quizizz during learning. Indicators of checking again cannot be seen in this study. This is because there is not enough time allocated to carry out these activities. The revised didactic design is then referred to as an empirical didactic design, which is classified as an epistemic learning pattern. This is because the didactic design is able to optimize mental and physical actions through epistemic steps when solving problems and constructing concepts.

Empirical didactic design can then be an alternative for mathematics teachers to help students learn triangular number patterns, especially when teachers want to optimize students' mathematical problem-solving abilities. Lesson study activities are also considered to be activities that teachers must continue to carry out to optimize teacher competence, which will automatically have an impact on improving the quality of mathematics learning. This empirical didactic design can also add evidence to the usefulness of epistemic learning patterns in mathematics learning. The hypothetical didactic design in this research is quite dependent on internet quotas, so it is not suitable for use in schools with poor internet connections. Therefore, future research should prepare a didactic design that can be implemented in schools with poor internet conditions so that the learning process becomes more effective.

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