

The concept of variable and the 3UV model: A literature review

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Abstract. Various difficulties have been documented in understanding the concept of an algebraic variable among secondary school, high school, and university students, such as not being able to differentiate the different uses of the variable, not being able to interpret the letter when it appears accompanied by a coefficient or exponent, and not recognizing the covariation of two related variables, among others. Given this problem, and based on a literature review covering the period 2005-2023, the objective of this article is to present the research trends on the concept of variable and the contributions for its study made through the theoretical framework called Model of the Three Uses of the Variable (Model 3UV). For this reason, a documentary research design was developed with a qualitative approach and at a descriptive level. The instruments for the collection and analysis of information were a search log, a bibliographic matrix, and a synthesis matrix, elaborated in the Excel software. The technique used to carry out the documentary review was the content analysis, from the deductive analytical categories: objective, research method, and conclusion. The advanced search was carried out in *Google Scholar* and of the total results obtained, the 10 most relevant articles published during the period 2005-2023 were selected. One of the main findings found was the identification of three trends in research with Model 3UV: the first, related to didactic teaching proposals; the second, concerning the understanding of difficulties in the learning of students; and the third, concerning teaching materials.

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INTRODUCTION

The development of algebraic thought in general and the understanding of the concept of variable in particular is fundamental to the study of other branches of mathematics and university careers. Hence, the study of elementary algebra has been included in the curriculum of secondary school education. However, various difficulties have been documented in the understanding of the concept of algebraic variable by students, from the secondary education level to the high school level, through the higher education environment (Carmo, 2019; Meléndez, 2015; Trigueros et al., 1996; Ursini & Trigueros, 2001).

According to Ursini et al. (2005), some of the difficulties that secondary school students face when approaching algebra are, for example: not being able to differentiate the different uses of the variable, not being able to interpret the letter when it appears accompanied by a coefficient or exponent, and not being able to recognize the covariation of two related variables, among other difficulties. In another study, Aguirre et al. (2018) confirmed that students in the second, third, and fourth year of secondary education, and not just basic education in Uruguay, make the mistake of simultaneously assigning different values to the same variable.

Faced with this problem during the teaching-learning process of school algebra, Ursini and Trigueros (2001) and Ursini et al. (2005) proposed the Model of the Three Uses of the Variable (Model 3UV) as an alternative to address the difficulties faced by students (Carmo, 2019). Such uses of the algebraic variable with their respective characteristics are as follows:

1. Variable as specific unknown

According to Ursini et al. (2005), its characteristic features are as follows:

I1: Recognize and identify, in a problematic situation, the presence of something unknown that can be determined considering the restrictions of the problem.

I2: Interpret the symbolic variable that appears in an equation, such as the representation of specific values.

I3: Replace the variable with the value or values that make the equation a true statement.

I4: Determine the unknown quantity that appears in equations or problems, performing algebraic operations, arithmetic, or both.

I5: Symbolize the unknown quantities identified in a specific situation and use them to pose equations.

2. Variable as a general number

According to Ursini et al. (2005), its characteristics are as follows:

G1: Recognize patterns and perceive rules and methods, in sequences and families of problems.

G2: Interpret the symbolic variable as the representation of a general, indeterminate entity that can assume any value.

G3: Deduce general rules and methods, in sequences and families of problems.

G4: Manipulate (simplify, develop) the symbolic variable.

G5: Symbolize general statements, rules, or methods.

3. Variable in a functional relation

According to Ursini et al. (2005), its characteristic features are as follows:

F1: Recognize the correspondence between related variables, regardless of the representation used (tables, graphs, verbal problems, analytical expressions).

F2: Determine the values of the dependent variable, given the values of the independent.

F3: Determine the values of the independent variable, given the values of the dependent.

F4: Recognize the joint variation of the variables involved in a functional relationship, regardless of the representation used (tables, graphs, verbal problems, analytical expressions).

F5: Determine the variation intervals of one of the variables, given the variation range of the other.

F6: Symbolize a functional relationship, based on the analysis of the data of a problem.

Therefore, the authors of Model 3UV consider it a useful theoretical-methodological tool to plan teaching in the classroom and to design student learning activities and diagnostic instruments (Ursini et al., 2005). Still, what have been the research trends on the concept of algebraic variable and the contributions to its study made through the 3UV Model?

To try to answer this question, the present work aims to present the research trends on the concept of algebraic variable and the contributions for its study made through the 3UV Model, during the period 2005-2023. To this end, the following sections describe the research method adopted, the results obtained, and the analysis carried out, as well as the conclusions obtained from the literature review carried out.

METHOD

Documentary research was carried out with a qualitative approach and at a descriptive level (Arias, 2012; Hernández et al., 2014). The search was carried out only in Google Scholar with the following Boolean equations and operators: "model 3UV", "uses of the variable" AND "model 3UV", considering the period from January 2005 to November 2023.

To select the most relevant papers from the total results obtained with the mentioned search equations, we proceeded to read the title and summary of the documents according to the following inclusion and exclusion criteria: (1) To be research articles and conference proceedings; (2) No degree thesis (bachelor's, master's, doctorate); and (3) No books or book chapters.

The data collection and analysis tools were a search log, a bibliographic matrix, and a synthesis matrix developed in Excel software. The technique used was the qualitative content analysis, made from the initial deductive categories described below in Table 1.

Table 1. Deductive categories of analysis

Category	Description
Objective	It refers to the objective with which each work was carried out to address some problems related to the concept of variable.
Method	It refers to the approach, design, level of depth, and subjects involved in each of the studies, as well as the instruments of collection and analysis of the information.
Conclusion	It refers to the conclusions and recommendations made in each of the studies.

Source: Own Elaboration

RESULTS AND DISCUSSION

The results obtained by searching Google Scholar were 211 documents in total. Therefore, the 10 most relevant articles were selected according to the inclusion and exclusion criteria mentioned in the previous section. This is shown in Table 2.

Table 2. Search results

Search Engine	Descriptors	Results
Google Scholar	"model 3UV"	202 (9)
Google Scholar	"uses of the variable" AND "model 3UV"	9 (1)

Note:

The total number of results is recorded, and the number of research reports included in the literature review is indicated in parentheses. Source: Own elaboration.

By date of publication, the 10 most relevant works selected, were located during the period 2005-2023 initially established, as can be seen in Figure 1.



Source: Own Elaboration

Figure 1. Documents by date of publication.

As can be seen in Figure 2, for the country where the selected studies were conducted, half of them were carried out in Mexico (Escalante & Cuesta, 2012; Herrera et al., 2016; Juárez, 2011; Rodríguez et al., 2023; Olea, 2007), the rest in Mexico-Brazil (López et al., 2010), Mexico-Italy (Ursini, 2019), Argentina (Marino & Isla, 2018), Brazil (Paula & Lima, 2017) and Spain (Pérez-

Martos & Moreno, 2023). This is explained by the country of origin of the creators of the 3UV Model.

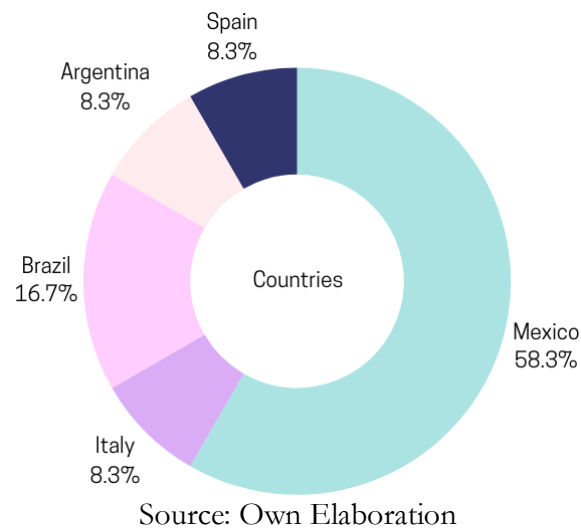


Figure 2. Documents by country.

The analysis performed by each of the deductive categories defined above is presented below.

A. Deductive analysis category: Objective

By analyzing the general objective and the specific objectives proposed in each of the selected articles, it was possible to identify three research trends on the concept of variable and the 3UV model related to the fields of teaching, learning, and teaching materials. This allowed us to achieve the general objective proposed when carrying out this work.

Trend related to the Teaching

In this first trend, some works related to proposals for didactic intervention were placed. Olea (2007), for example, in his work with Mexican high school students, proposes the use of technology, specifically the electronic spreadsheet, to support understanding of the concept of variable and the consequent development of algebraic thinking.

For their part, Marino and Isla (2018), presented the use of the Portfolio as a formative evaluation tool to encourage students to metacognize the three uses of the variable, from self-reflection, because traditionally in mathematics class the problem-solving and exercises are privileged, and there are no spaces for metacognitive reflection about errors and difficulties.

Finally, we find the study by Rodríguez et al. (2023), who proposed to analyze the effect of the implementation of didactic units, designed based on the 3UV model, on the pre-algebraic thinking of Mexican primary school students. The results showed significant progress in the resolution of activities related to the three uses of the variable.

Trend in reference to learning

In this second trend, is located most of the research related to difficulties in the learning and understanding of students, and even teachers, on the three uses of the variable.

One of the works is that of López et al. (2010), who carried out a comparative study between Mexican and Brazilian students at the high school level to know if there was any difference in the recognition of the three uses of the variable, with the result that there is no significant difference.

Another interesting study is that of Juárez (2011), who analyzed the difficulties that Mexican teachers of mathematics at the secondary school level presented when answering a questionnaire in which the three uses of the variable were addressed. The results showed a low level of understanding of the concept of variable by the participating teachers, which suggests that it could explain the difficulties of students when studying algebra.

In the higher educational level is the work of Escalante and Cuesta (2012), who analyzed the difficulties faced by Mexican students of Economics and Informatics in understanding the concept of variable. The results showed that university students did not have a greater understanding of the variable than did high school students; moreover, they resorted to arithmetic procedures for solving the problems posed.

For their part, Herrera et al. (2016) proposed to describe and analyze the difficulties that Mexican high school students showed when trying to solve problems related to generalization and modeling processes. They found evidence that there is no understanding of the concept of variable and that students' knowledge is of procedural and arithmetic type.

Years later, Ursini (2019), one of the creators of the 3UV model, carried out comparative research similar to that of López et al. (2010), but among Mexican and Italian high school students with a double objective: identify differences and similarities between the groups studied, and analyze the understanding of the concept of variable in year three students. The results showed that many of the misinterpretations, errors, and difficulties regarding the uses of the variable can be avoided, as they derive from the school-based approach of elementary algebra used in each country.

Finally, one of the most recent studies is that of Pérez-Martos and Moreno (2023), who were interested in observing and describing the interpretation and representations used by a group of primary school students in Spain, to represent the variable in functional relation when solving a function comparison task. The results showed an approach of the students to the concept of domain, the forms of representation (symbolic, pictorial, and verbal), and interpretation (identification of the independent and dependent variable, as well as covariation and correlation) associated with the concept of variable in functional relation.

Trend referred to Teaching materials

Finally, in the third trend was located the study of Paula and Lima (2017) who, from the 3UV model, conducted a content analysis to learn how the concept of variable was addressed in textbooks used at the primary school level in the Brazilian education system. This was intended to encourage a critical look at teachers who use these books as teaching materials in their mathematics classes.

As has been described so far, when analyzing the objective with which the 10 selected studies were carried out, three research trends were identified around the 3UV Model: one oriented to the design of didactic teaching proposals, another focused on identifying errors and difficulties of the students, and one more related to the analysis of teaching materials such as textbooks.

B. Deductive analysis category: Method

The following describes the approach, design, depth level, and subjects involved in each of the selected studies, as well as the instruments for collecting and analyzing the information used in them.

Subcategory: Focus on research

As can be seen in Figure 3, the research approaches adopted in the selected research were qualitative, quantitative, and mixed. The qualitative studies were those of Pérez-Martos & Moreno (2023), Paula & Lima (2017), Herrera et al. (2016), Escalante & Cuesta (2012), and Marino & Isla (2018). While Juárez (2011), Rodríguez et al. (2023), Olea (2007), and Ursini (2019) conducted studies with a mixed approach, only López et al. (2010) adopted the quantitative approach to their work.

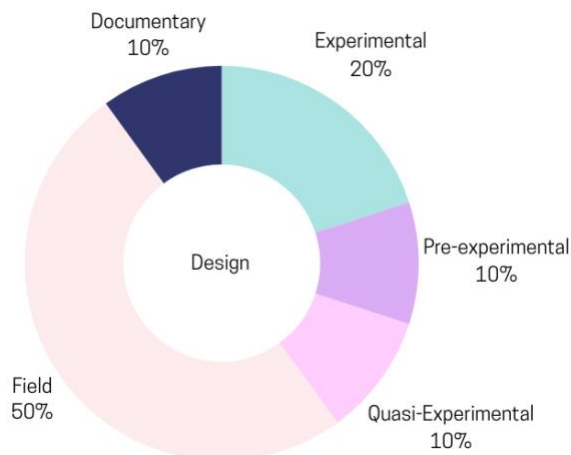


Source: Own Elaboration

Figure 3. Distribution of articles according to research approach.

Subcategory: Design of research

As can be seen in Figure 4, the research design identified in the selected articles corresponded to the specific needs of each study. For example, Escalante & Cuesta (2012) and Pérez-Martos & Moreno (2023) adopted an experimental design; Rodríguez et al. (2023) and Olea (2007) used pre-experimental and quasi-experimental designs, respectively. In the cases of Herrera et al. (2016), Juárez (2011), López et al. (2010), Marino & Isla (2018), and Ursini (2019), they followed a field design; and only the work of Paula & Lima (2017) was documentary.

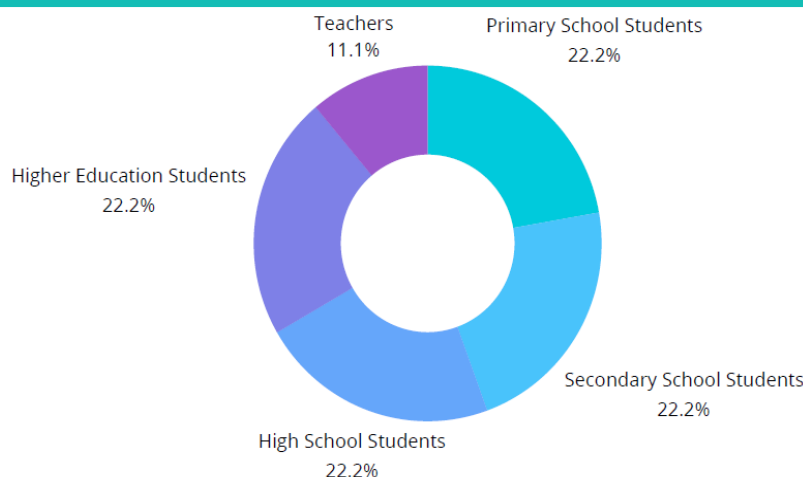


Source: Own Elaboration

Figure 4. Distribution of articles according to their research design.

Subcategory: Participants and participants

Without considering the work of Paula & Lima (2017), since they carried out an analysis of textbooks, the subjects who participated in eight of the selected studies were primary school students (Rodríguez et al., 2023; Pérez-Martos & Moreno, 2023), secondary school students (Ursini, 2019; Olea, 2007), high school students (Herrera et al., 2016; López et al., 2010), and higher education students (Escalante & Cuesta, 2012; Marino & Isla, 2018), and only one in which teachers participated (Juárez, 2011). This can be seen in Figure 5.

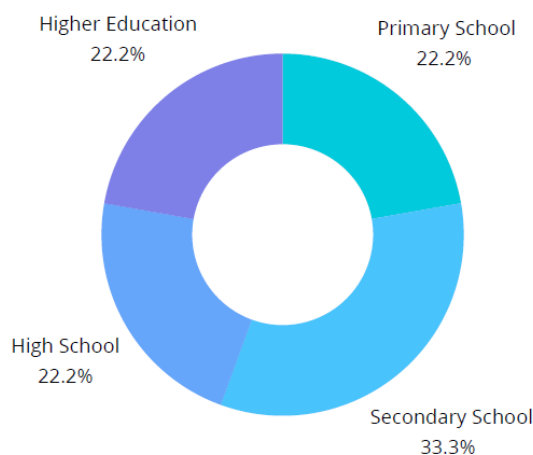


Source: Own Elaboration

Figure 5. Distribution of articles according to the participants.

Subcategory: Education level

As can be seen in Figure 6, the education levels at which the selected studies were conducted range from primary to higher education. The work of Rodríguez et al. (2023) and Pérez-Martos & Moreno (2023) was carried out at the primary education level, providing valuable insights on the learning processes in the initial stages of education. While in the secondary school level are the studies of Olea (2007), Juárez (2011) and Ursini (2019). At the high school level are the investigations of Herrera et al. (2016) and López et al. (2010). At the higher education level are located the works of Escalante and Cuesta (2012), and Marino & Isla (2018).



Source: Own Elaboration

Figure 6. Distribution of articles according to the education level where the study was conducted.

Subcategory: Instruments of information collection

The data collection tools used in the studies analyzed are diverse and adapted to the specific needs of each research. The study by Rodríguez et al. (2023) and Olea (2007) used pre-test and post-test, in addition to the interview in the second case. López et al. (2010) and Ursini (2019) used the questionnaire to conduct their comparative studies. Meanwhile, Herrera et al. (2016) and Juárez (2011), in addition to the questionnaire, applied interviews to conduct their studies. Escalante & Cuesta (2012) also implemented interviews and a written test. Finally, Pérez-Martos & Moreno (2023) used a written task, work cards, written cardboard, video recordings, and audio recordings.

However, the various approaches (qualitative, quantitative, and mixed) adopted in the studies analyzed, as well as the education level of the participating subjects where they were carried

out (primary school, secondary school, high school, and university) and the research design, the results found lead to the same conclusions regarding the contributions of the 3UV Model for the teaching and learning of the concept of algebraic variable.

C. Deductive analysis category: Conclusion

Here are mentioned the most relevant conclusions reached by the authors of the selected studies, according to the research trends described above in the first deductive category of analysis.

Trend related to the Teaching

Olea (2007) concludes that the use of technology, such as the electronic spreadsheet, favors teachers' teaching methods so that students understand various aspects of the uses of the algebraic variable.

For their part, Marino & Isla (2018) concluded that the reflection prompted by having implemented the Portfolio as an evaluation instrument allowed them to develop metacognition with students about their errors and difficulties in understanding the concept of variable.

Meanwhile, Rodríguez et al. (2023) declare that they have detected significant progress in primary school students when they solved activities related to the uses of the variable as an unknown, general number, and functional relationship.

Trend related to Learning

López et al. (2010), when comparing the performance of Mexican and Brazilian students at the high school level, conclude that they have difficulties characterizing the concept of variable, and therefore point to the need to include such characterization in the contents of the school curriculum of that education level.

Juárez (2011), after conducting a study with Mexican teachers of secondary school level who showed difficulties in understanding the three uses of the variable, suggests the conduct of teacher training workshops on the 3UV model, as well as continued research in this area taking into account the gender of the teachers.

Meanwhile, Escalante & Cuesta (2012) investigating errors and difficulties in understanding the concept of variable faced by Mexican university students, concluded that they have failed to develop algebraic thinking, despite having studied mathematics at the secondary school and high school level.

Herrera et al. (2016) concluded that Mexican high school students do not understand the concept of variable, since their knowledge is procedural and arithmetic, with a low level of reasoning that does not allow them to relate concepts.

On the other hand, Ursini (2019), after conducting a comparative study among Mexican and Italian high school students, concluded that the errors and difficulties they face when working with the variable are explained by the way of approaching elementary algebra in each country, therefore recommends revising their curricula and developing more comparative studies.

Finally, Pérez-Martos & Moreno (2023) concluded that primary school students are capable of thinking algebraically and functionally, so they concur with the proposal of early algebra.

Trend related to Didactic Materials

In their work, Paula & Lima (2017) concluded that the 3UV model is a useful tool to improve teaching practice, as a diagnostic tool that allows teachers to identify errors and difficulties of students and thus, promote the teaching of elementary algebra.

As has been described in this category about some of the most relevant conclusions reported in the studies analyzed, the 3UV model has served to identify errors and difficulties in the learning of secondary school students (Ursini, 2019), high school students (Herrera et al., 2016; López et al., 2010) and university students (Escalante & Cuesta, 2012), as well as that it is possible to introduce algebra at the primary school level (Pérez-Martos & Moreno, 2023). Likewise, it has allowed teaching proposals to be made that improve the understanding of the concept of algebraic

variable, through the use of the spreadsheet (Olea, 2007), the portfolio (Marino & Isla, 2018), and the resolution of activities (Rodríguez et al., 2023).

CONCLUSIONS

After performing an advanced search in Google Scholar on the 3UV Model, during the period 2005-2023, 10 relevant research articles were selected from a total of 211 results obtained. Most of the selected studies were conducted in Mexico and the rest in Argentina, Brazil, Spain, and Italy. The analysis of the work was carried out from the deductive categories: objective, method, and conclusion, defined previously. In terms of the stated objective in each of the studies, three research trends were identified that have been developed. The first concerns proposals for teaching the concept of variable; the second relates to difficulties in the learning of students; and the third relates to teaching materials such as textbooks.

The research trends identified coincide with those suggested by Ursini & Trigueros (2001) more than two decades ago, in terms of using the 3UV Model, as a guide to design diagnostic analysis tools of students' knowledge of the different uses of the variable, to design teaching activities, and to analyze teaching materials. This, according to Sánchez (2017), allows us to link professional teaching practice with the findings obtained in research on the teaching-learning process of school algebra. The model studied serves well as a guiding basis for teachers to analyze students' algebraic tasks (know and assess their understanding), as well as to improve their pedagogical practice.

In that same sense, Kú-Euán et al. (2021) suggest the creation of Professional Development spaces in which teachers and researchers interact to reflect on how to address students' difficulties in learning the concept of variable, based on the results of research in Mathematics Education. Regarding the research method adopted in the analyzed works, the qualitative approach predominates, followed by the mixed, and the quantitative. As for the participants, except for one study in which secondary school teachers participated, the rest were students of all education levels (primary, secondary, high school, and university).

Finally, based on the main conclusions drawn from the studies analyzed, it is necessary to highlight the importance of continuing to use the 3UV Model as a theoretical-methodological framework to identify difficulties in students' understanding and learning (Escalante & Cuesta, 2012; Herrera et al., 2016; López et al., 2010; Paula & Lima, 2017; Ursini, 2019), and even from teachers (Juárez, 2011) about the concept of algebraic variable, as well as to design didactic teaching proposals (Rodríguez et al., 2023; Marino & Isla, 2018). This is aimed at improving the teaching practice and the teaching-learning process of the concept of variable, to promote the development of algebraic thinking in students from the first years of schooling (Pérez-Martos & Moreno, 2023). In addition to the latter, Ardiansari et al. (2022) suggest that an adequate understanding (as a relational symbol) of the equal sign from primary school is essential to support algebraic thinking skills.

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