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The teaching-learning process of the continuity of real variable functions: A literature review

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Abstract.
This article´s objective is to present the current state of research on the teaching-learning process of continuity of real variable functions. To achieve this, a documentary research design with a qualitative approach and a descriptive depth level was adopted. The instruments for data collection and analysis included a search log, a bibliographic matrix, and a synthesis matrix, created using Excel software. The technique used for the literature review was qualitative content analysis, based on deductive analytical categories: objective, theoretical framework, research method, and conclusions. Advanced searches were conducted on Google Scholar, on Scopus and Web of Science and out of the total results found, 13 relevant articles published during the period 2020-2023 were selected. Among the main findings, the use of a variety of theoretical frameworks in the foundation of the analysed studies was identified, such as the theory of semiotic representations, the MTSK model, the TPACK, and the conceptual contributions of Tall and Vinner, among others. Additionally, a growing interest in the use of technological tools, such as GeoGebra, to enhance the teaching and learning process of mathematics in general, and the continuity functions in particular. Although its limitations, it was identified that this software can be used as a tool for visualization, construction, representation and communication of mathematical knowledge.

Keywords:
Continuity of functions; real variable; teaching; learning; literature review

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INTRODUCTION

In Mexico, the study of the continuity concept is framed within Differential and Integral Calculus, both at the secondary education and university levels, as it is considered necessary to understand topics such as the analysis of variable variation in relation to limits, derivatives, graphical representation, and optimization. It is also crucial in studying areas related to integration and optimization, defining new functions such as those defined through integrals and pointwise functions constructed via limits, as well as in many other areas like differential and partial differential equations and integral equations. However, the teaching and learning of calculus in general, and continuity in particular, pose a series of difficulties for students and teachers, as they require a higher level of abstraction, argumentation, and demonstration (Lan & Zhou, 2020; Morales et al., 2021; Morales & Damián, 2020).

A primary factor explaining these difficulties lies in the treatment order given to continuity in the curriculum is limited as it is found in only one objective of the study program (SEP, 2018). This indicates that its treatment is both restricted and superficial. Additionally, in school textbooks,

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starting with an intuitive idea of the concept, followed by the formal definition and the study of some examples (Sulastri, 2023). In this regard, Morales and Damián (2020) point out a lack of analytical activity regarding the conditions and properties of mathematical objects (such as continuity at a point), as knowledge is presented as a finished product.

A second element explaining difficulties in teaching and learning continuity is related to comprehension problems that often arise due to differences between the student's concept image and the formal concept definition. Students may struggle to reconcile their intuition and informal understanding with mathematical notation and formality (Hanke & Schäfer, 2017; Tall & Vinner, 1981). Some studies have focused on the idea that informal treatments of these concepts lead students to develop misconceptions (Ferrini-Mundy & Graham, 1994). Others have reported epistemological obstacles such as distinguishing between discreteness and continuity, the convergence of the infinite with the indivisible, and the differentiation and acceptance of actual and potential infinities (Hernández-Suárez, 2017; Trujillo, 2019; Crespo, 2004; Hitt, 2003).

Finally, a third element explaining these difficulties is linked to the conceptions that teachers have about continuity, and pass on to their students (Aparicio & Cantoral, 2007). In this regard, Delgado (2013) explains that the presence of misconceptions among mathematics teachers about the concept of continuity of a function at a point has been identified, often influenced by the use of inadequate exemplary images. It is acknowledged that, although teaching practices may be correct from an algorithmic and rigorous perspective, students' difficulties in understanding pointwise continuity may be linked to these teacher-held conceptions. This represents a significant barrier to learning.

Therefore, the aim of this work is to understand the current state of research on the teaching-learning process of the topic: continuity of real-variable functions. The relevance of this study lies in the absence of similar literature reviews that have analyzed this particular topic. Thus, one of the main contributions of this work is to provide mathematics teachers with a general overview of the most recent research findings, which can help improve their pedagogical practices and address students' learning difficulties.

The research method adopted will be described next, followed by the presentation of results and the conducted analysis, and finally, some general conclusions derived from the literature review will be included.

METHOD

To achieve the aforementioned objective, a documentary research design with a qualitative focus and a descriptive depth level was adopted (Arias, 2012; Hernández et al., 2014). The methodological procedure followed two general phases. The first, of a heuristic nature, involved the search and selection of documentary sources of information. The second, of a hermeneutic nature, focused on the reading and analysis of these sources.

The heuristic phase focused on the Google Scholar search engine and the Scopus and Web of Science databases. Table 1 presents the equations and Boolean operators used for the advanced search conducted in both Spanish and English during the period from January 2020 to October 2023.

Table 1. Results by equation, search engine and database

Search engine/database	Search Equation	Results
Google Académico	"Continuidad de funciones"	254
	"Continuidad de funciones" AND "profesores" AND "estudiantes"	177
	"Continuidad de funciones" AND "estudiantes" AND "aprendizaje" AND "bachillerato"	65
	"Continuidad de funciones" AND "profesores" AND "estudiantes" AND "aprendizaje" AND "enseñanza" AND	55

Search engine/database	Search Equation	Results
	"bachillerato"	
	"Continuity of functions" AND "learning" AND "students" AND "high school"	65
	"Continuity concept" AND "learning" AND "students"	77
Scopus	"Continuity" AND "students"	37
Web of Science	"Continuity of functions"	1
	Total	731

Source: Own work

Following this, the titles and abstracts of the initial results were reviewed, selecting only 13 documents deemed most relevant and meeting the following inclusion criteria: being research articles and addressing the teaching-learning process of continuity in real-variable functions. Dissertations, books, book chapters, and curricula, which also appeared among the total results, were excluded. Table 2 lists the authors of the selected articles by search engine and database reviewed.

Table 2. Selected articles

Search engine/database	Selected Articles
Google Académico	Branchetti et al. (2020), Budak & Akcay (2022), Fonseca & Henriques (2020), Laderas et al. (2022), Morales et al. (2021), Morales & Damián (2020), Fernández et al. (2023), Tong et al. (2021), Valongo y Felgueiras (2022), Vargas y Murcia (2022)
Scopus	Rodríguez-Nieto et al. (2022), Safarini et al. (2023)
Web of Science	Munyaruhengeri et al. (2023)

Source: Own work

The instruments for data collection and analysis were developed in Excel software, including a search log that recorded the date, search equation, total results, and most relevant results; a bibliographic matrix containing author names, year of publication, document title, country, and language; as well as a synthesis matrix used to concentre and analyse information of the selected articles according to the defined deductive analysis categories system outlined in Table 3. The bibliographic matrix and the synthesis matrix, also known as the analytical content matrix, are recommended as useful tools for documentary research (Gómez et al., 2015). In the synthesis matrix, the analysis categories (vertically) and the selected articles (horizontally) were recorded, facilitating both linear and cross-sectional analysis.

Table 3. System of deductive analysis categories

Category	Description
Objective	It refers to the objective for which the study was conducted to address an issue related to the continuity of functions.
Theorical Framework	It refers to the theories and/or concepts on which the analysis of the continuity of functions was based.
Method	It refers to the approach, design, depth level, and participants with whom the study was conducted, as well as the instruments for data. collection.
Conclusion	It refers to the conclusions and suggestions derived from the results analysed in each study.

Source: Own work

The technique used during the hermeneutic phase was content analysis, understood as "a set of procedures aimed at producing an analytical meta-text that represents the textual corpus in a transformed manner" (Díaz & Navarro, 2007). To ensure greater objectivity in the study, triangulation was conducted in the analysis and interpretation of the information by the first two authors of this work. This means that the second author also reviewed the selected articles based on the deductive categories defined in Table 1 and examined the information recorded by the first author in the synthesis matrix. To resolve discrepancies, both authors cross-checked the entries in the matrix with the content of the analyzed articles and reached a final agreement.

RESULTS AND DISCUSSION

The following are the findings found in the 13 selected documents, according to each of the analysis categories described above. Firstly, it should be considered that, regarding the location where each of the studies was conducted, the majority were situated in the Americas (eight), with the remainder in Europe (two), Asia (two), and Africa (one). Specifically, by country, three studies were conducted in Mexico (Morales et al., 2021; Morales & Damián, 2020; Rodríguez-Nieto et al., 2022), two in Peru (Laderas et al., 2022; Fernández et al., 2023), one in Colombia (Vargas & Murcia, 2022), one in Brazil (Fonseca & Henriques, 2020), one in the United States (Budak & Akcay, 2022), one in Portugal (Valongo & Felgueiras, 2022), one in Italy (Branchetti et al., 2020), one in Vietnam (Tong et al., 2021), one in Indonesia (Safarini et al., 2023) and one more in Rwanda (Munyaruhengeri et al., 2023).

Deductive analysis category: Objective

When considering the research objective of all the selected articles, two general trends could be identified. The first trend is related to learning about the object of study in question. Here, the work of Fonseca and Henriques (2020) is situated, as they were interested in understanding the meanings that Brazilian teachers in training attribute to the concept of continuity of a function. Similarly, there is the study by Budak and Ozkan (2021), who aimed to analyse the thought processes of American teachers to identify differences between the personal concept and the formal definition of the continuity concept. The study by Safarini et al. (2023) is also included, which investigates the results of students' proceptual thinking in learning the concept of differentiability using Classroom Activities in Desmos (DCA) based on the Three Worlds of Mathematics (TWM) framework. Finally, within this trend, the work of Rodríguez-Nieto et al. (2022) is positioned, whose objective was to characterice the quality of the intra-mathematical connections made by Mexican university students when solving tasks on the derivative.

The second identified trend refers to the teaching of continuity. It includes works that report results from experiences and pedagogical proposals for teaching the continuity of functions. Nine articles from the selected ones were placed within this trend. Of them, the majority are of an empirical nature (Morales & Damian, 2020; Branchetti et al., 2020; Laderas et al., 2022; Tong et al., 2021; Vargas, 2022; Fernández et al., 2023; Munyaruhengeri et al., 2023, Valongo & Felgueiras, 2022), and one is of a theoretical-didactic nature (Morales et al., 2021), but in general, they mainly coincide in the interest in studying the impact of the use of technologies, particularly the dynamic software GeoGebra, in teaching the continuity of functions.

Deductive analysis category: Theoretical Framework

When analysing the theoretical framework employed in the selected articles, a variety of theories and concepts were found to have been used to study continuity teaching- learning process, as can be seen in Table 4.

So, the works of Morales and Damián (2020) and Branchetti et al. (2020) coincide in the use of Duval's Semiotic Representations. The remaining eleven articles employed different theoretical frameworks. However, in some, such as in the case of Branchetti et al. (2020), they also use Tall and Vinner's (1981) Concept Images and Concept Definitions, as employed by Budak and Akcay (2022) in their study 'Pre-Service Teachers' Understanding of Continuity.

Table 4. Theoretical frameworks used in the selected studies

Paper	Theorical Frame
Morales & Damián (2020)	Semiotic Representations Theory
Branchetti et al. (2020)	
Fonseca & Henriques (2020)	Instrumental Understanding and Relational Understanding
Morales et al. (2021)	Mathematics Teacher's Specialized Knowledge (MTSK)
Tong et al. (2021)	Discovery Learning
Budak & Akcay (2022)	Conceptual Image, Formal Definition of the Concept, Personal Definition of the Concept, and Conflict Factors (Tall and Vinner)
Laderas et al. (2022)	Use of Information and Communication Technologies
Rodriguez-Nieto (2022)	The typology of mathematical connections and the quality levels of mathematical connections
Vargas (2022)	The Disciplinary Component (learning calculus), the Pedagogical Component (meaningful learning, multiple intelligences), the Human Development Component (capabilities theory), and the Technological Component (Implementation of ICT, transmedia narrative)
Valongo y Felgueiras (2022)	Abstraction in Context RBC R-action, B-action, and C-action
Fernández et al. (2023)	Mathematical Workspace (MWS)
Munyaruhengeri et al. (2023)	TPACK: Technology, Pedagogy, and Content Knowledge — enhanced by Technological Pedagogical Knowledge (TPK), Content Knowledge (CK), and Technological Content Knowledge (TCK)
Safarini et al., (2023)	Three worlds of the mathematics (TWM)

Source: Own work

Deductive analysis category: Method

Here, the approach, design, participants, educational level, and instruments of the selected studies are described. Of the 13 articles analysed, it was identified that the majority (eight) of them adopted a qualitative approach, while the rest followed a quantitative approach (four) and a mixed approach (one), as shown in Figure 1.

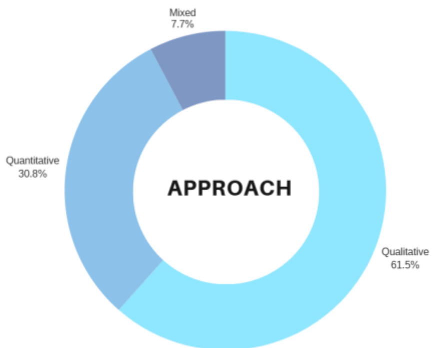

Figure 1. Methodological approach (Source: Own work)

The articles that adopted the qualitative approach are those of Morales and Damián (2020), Fonseca and Henriques (2020), Morales et al. (2021), Budak and Akcay (2022), Valongo and Felgueiras (2022), and Fernández et al. (2023), Safarini et al., (2023) and Rodríguez-Nieto et al. (2022). The four that used the quantitative approach are those of Branchetti et al. (2020), Laderas et al. (2022), Vargas (2022), and Munyaruhengeri et al. (2023). Finally, the article by Tong et al. (2021) is the only one that employed the mixed approach.

Regarding the methodological design adopted in the 13 analysed articles, the following was found: Descriptive 3, Interpretative 4, Quasi-experimental 2, Experimental 1, Theoretical-Didactic 1, Multiple Case Study 1 and Case Study 1. This can be observed in Figure 2.

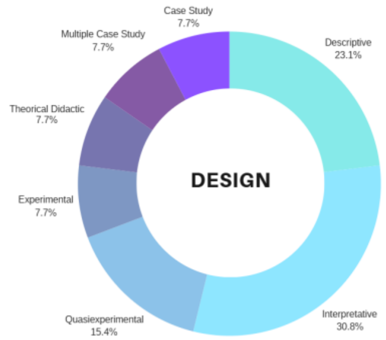

Figure 2. Methodological design (Source: Own work)

The articles with a descriptive design are those of Morales and Damián (2020), Branchetti et al. (2020), and Tong et al. (2021). The four with an interpretative design are those of Fonseca and Henriques (2020), Valongo and Felgueiras (2022), Fernández et al. (2023) and Safarini et al., (2023). The ones with a quasi-experimental design are those of Vargas (2022) and Munyaruhengeri et al. (2023). Laderas et al. (2022) present an experimental design. The Theoretical-Didactic design is found in Morales et al. (2021), the Multiple Case Study design is in Budak and Akcay (2022) and Case Study Rodríguez-Nieto et al. (2022).

As for the participating subjects, in 12 of the studies analysed, students from Upper Secondary Education (pre-university) and higher education participated. In the teacher category, only two were counted; in the first, teachers from the pre-university level participated, and in the second, they were pre-service teachers. This can be observed in Figure 3.

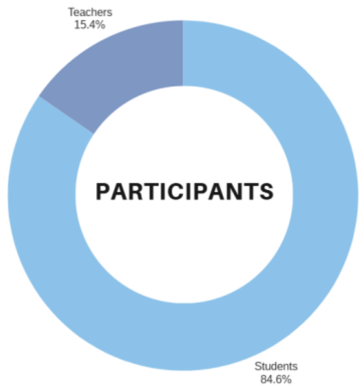

Figure 3. Participants (Source: Own work)

The articles focused on pre-university students are those of Morales and Damián (2020), Branchetti et al. (2020), Fonseca and Henriques (2020), Tong et al. (2021), Laderas et al. (2022), Vargas (2022), Valongo and Felgueiras (2022), Fernández et al. (2023), Munyaruhengeri et al. (2023), Safarini et al. (2023) and Rodríguez-Nieto et al. (2022). The articles in which teachers participated are those of Morales et al. (2021) and Budak and Akcay (2022). This last finding highlights the lack of studies involving teachers. It suggests the need for more research with teachers, as Delgado (2013) has reported the existence of misconceptions among mathematics teachers regarding the concept of continuity of a function at a point, and it is possible that students' difficulties in understanding pointwise continuity are related to these conceptions.

Considering the education level where the studies were conducted, it was found that seven of them were carried out at the pre-university level (Morales & Damián, 2020; Branchetti et al., 2020; Fonseca & Henriques, 2020; Morales et al., 2021; Tong et al., 2021; Fernández et al., 2023; Munyaruhengeri et al., 2023), and six at the higher education level (Budak & Akcay, 2022; Laderas et al., 2022; Vargas, 2022; Valongo & Felgueiras, 2022; Safarini et al., 2023; Rodríguez-Nieto et al., 2022) as can be seen in Figure 4.

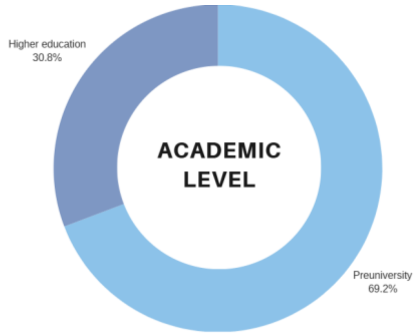

Figure 4. Educational level (Source: Own work)

Concerning the instruments used, seven articles employ a didactic proposal (Morales & Damián, 2020; Branchetti et al., 2020; Fonseca & Henriques, 2020; Morales et al., 2021; Vargas, 2022; Fernández et al., 2023; Safarini et al., 2023). The remaining six studies use various other instruments. In the case of Tong et al. (2021), they conducted a design study, Budak and Akcay (2022) designed and presented a problem, Laderas et al. (2022) used a pre-test and post-test, Valongo and Felgueiras (2022) applied tasks, Munyaruhengeri et al. (2023) administered an achievement test and Rodríguez-Nieto et al. (2022) designed and applied an eight-task protocol. This can be observed in Figure 5.

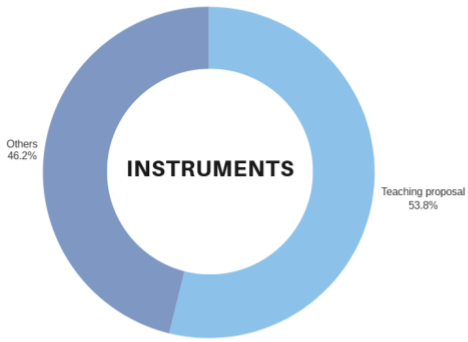

Figure 5. Instruments (Source: Author's own work)

Deductive analysis category: Conclusions

Based on the conclusions and suggestions proposed in each of the studies, the following emerging subcategories of analysis were identified: similarities, differences, and recommendations made by the authors of the selected works.

Emerging subcategory of analysis: Similarities

The articles titled 'Didactic Strategy to Introduce the Concept of Punctual Continuity with Pre-University Students' and 'Alternative Didactic Approach to Introduce the Concept of Punctual Continuity to Pre-University Teachers' are investigations conducted by the same authors (Morales & Damián, 2020) in subsequent years. They conduct their studies first with students and later with teachers, both at the same education level (pre-university), reaching similar conclusions regarding the importance of creating the right circumstances to allow for the formulation of the definition, rather than presenting the concept as something already finalised. They emphasise that their proposal modifies the conventional structure of how information is presented in textbooks intended for the pre-university level.

On the other hand, Branchetti et al. (2020), Tong et al. (2021), and Fernández et al. (2023) conclude that their respective proposals achieved a positive impact on students' learning of continuity, prompting the need to continue exploring and expanding research. For example, Branchetti et al. (2020) observed that the application of the topological approach was suitable for improving students' ability to use graphical representations in expressing the continuity of functions. While Tong et al. (2021) mention that the results obtained in the conducted experiment indicated that instruction on continuous functions when using GeoGebra software, has a positive impact on students' attitudes, motivation to learn, and academic achievements in mathematics. Finally, Fernández et al. (2023) explain that the task favours the development of the student's Mathematical Workspace (ETM) through the use of a GeoGebra applet. Besides, the study by Safarini et al. (2023) concluded that the DCA based on the TWM framework could be used as a generic organizer to build students' perceptual understanding of continuity and differentiability, as well as their proceptual thinking and graphical representation. Thus, it has been shown that the DCA, based on the TWM framework, functions as a flexible organizer to support the development of students' perceptual understanding of continuity and differentiability, along with their proceptual thinking and graphical interpretation skills. The results of this research validate that the DCA, utilizing TWM as a general organizer, effectively fosters students' proceptual thinking abilities during their learning process of differentiability.

Another study reporting the use of technological tools is that of Munyaruhengeri et al. (2023), who specifically use GeoGebra. In this regard, use it to enhance students' achievements in learning limits and continuity of functions. They compare the results when using dynamic software and the traditional paper-and-pen method. They conclude that their study has confirmed the effectiveness of using GeoGebra to achieve learning of limits and continuity of functions. However, it has been observed that the traditional approach using paper and pen also results in learning achievements, although the use of GeoGebra software is associated with greater success in both procedural and conceptual understanding of the concept.

Similarly, Laderas et al. (2022) and Vargas (2022) agree that strategies employing technology enable discussions on inclusion, given that ICTs are not within reach of all students, starting with the limitations institutions face regarding internet service. Despite this, they argue that the use of technology contributes to making concepts in the fields of science and engineering more dynamic and accessible.

The results found in this subcategory align with the studies conducted by Thurston (1994), Zengin (2018), and Villa-Ochoa et al. (2018) regarding the importance of using technology in the teaching-learning process of mathematics. The former emphasizes the transformative impact of technology, affecting both the methodology and content of instruction in the discipline (Thurston, 1994). Zengin (2018) highlights that the dynamic features of GeoGebra create an interactive, visual, and flexible learning environment, utilizing tools like sliders and the drag-and-drop function. Additionally, one of the reasons for its widespread use in the classroom is its simple interface and its status as free software, available as a mobile application. Finally, Villa-Ochoa et al. (2018) emphasize the importance of creating educational environments that incorporate digital technologies, as these facilitate the construction of multiple meanings and enhance the understanding of mathematical objects related to calculus in various contexts. However, regarding limitations, they note that its use requires spaces equipped with computers, a feature that is difficult to meet in many public institutions in Mexico.

Emerging subcategory of analysis: Differences

Fonseca and Henriques (2020) conclude that students, in general, demonstrate a relational understanding of the concept of continuity of a function at the end of the educational experience. They can establish connections between various meanings and representations of the concept, using these skills to address related problems. Despite these achievements, some students still maintain an instrumental understanding, evidencing deficiencies in constructing clear representations of the concept of continuity. These deficiencies are attributed to the lack of geometric, algebraic, or verbal rigour in the expression of these representations.

In the case of Budak and Akcay (2022), they point out that their analyses showed that the PSTs interviewed had an incomplete understanding of continuity. They conclude that both educators and teacher educators should create mathematical environments that enable students and future teachers to be aware of the discrepancies between their individual conceptual perceptions and the formal definitions of mathematical concepts and principles.

The research by Rodríguez-Nieto et al. (2022) revealed that university students rely more on graphical and algebraic processes, neglecting the formal arguments (theorems and definitions) that support analytical processes in solving a mathematical task, thanks to the characterization of the quality of mathematical connections. This situation led them to conclude that the quality of mathematical connections is determined based on the processes and mathematical arguments used by the student, considering that a connection exists when the answer is consistent with what is mathematically accepted; otherwise, it is said that the student does not make mathematical connections.

Finally, Valongo and Felgueiras (2022) investigated the notion of continuity construction in university students, employing abstraction in context as the theoretical framework. Considering epistemic actions, it can be concluded that the process of abstraction was constantly triggered through Action R, as students recognised the need to employ previous constructions (such as quadratic and piecewise functions, domain, and neighbourhood, among others). Later, the students used these previous constructions to obtain intermediate solutions, such as graphical representations and domain definitions, which are linked to Action B. Finally, Action C was always initiated by the reorganisation of the constructions developed in Action B. Suggesting that future work should focus on consolidation (Co-Action).

Emerging subcategory of analysis: Recommendations

Morales et al. (2021) recommend that teachers conduct a thorough analysis of the theoretical and methodological framework adopted in their proposal, as well as make adaptations according to the characteristics of the students, organise, develop, and implement the suggested activities. Simultaneously, it is advised to carry out experiments with the proposal alongside students and compare the results obtained with the theoretical projections made by the experts who validated their proposal, with the aim of refining aspects of it.

Therefore, Rodríguez-Nieto et al. (2022) explain in future research the category of metaphorical connections can be used in studies focused on both the student and the teacher of mathematics.

Finally, Vargas (2022) suggests continuing to apply the transmedia tool (Pixton) not only in the final grades of the school year in basic secondary or middle school education but also exploring its application in other areas of mathematics, such as metric and random geometry. This practice benefits the development of mathematical concepts in students with Special Educational Needs (SEN), generating a positive impact on inclusion processes.

CONCLUSIONS

Based on the advanced search conducted on Google Scholar, Scopus and Web of Science on the learning and teaching of continuity of functions during the time period from January 2020 to October 2023, 13 relevant research articles were selected from the total results obtained. Most of them were conducted in countries in America, and the rest in Europe, Asia, and Africa.

Considering the objective of the selected works, two general trends in research were identified: one focused on teaching and the other on learning the continuity of functions. Regarding their theoretical foundation, a diversity of theories and concepts was found in the studies, such as the theory of semiotic representations, the MTSK model, TPACK, discovery learning, and contributions from Tall and Vinner (1981), among others.

In most of the works, a qualitative approach was adopted, followed by the quantitative and mixed approaches, with the participation of mainly secondary education students. Regarding the main findings, there was a growing interest in the use of technological tools, such as GeoGebra, to enhance the teaching and learning process of continuity of functions. In this regard, it has been documented that the use of dynamic geometry software such as GeoGebra can enhance and develop students' competencies and skills, such as critical, creative, and innovative thinking, promoting problem-based learning as a methodology aligned with 21st-century educational approaches (Kim, 2017). In teaching and learning situations, the software can be used as a tool for visualization, construction, representation, and communication of mathematical knowledge (Prieto, 2016; Yohannes & Chen, 2021).

Undoubtedly, GeoGebra is an appropriate teaching resource for improving student participation and performance in selected mathematics topics in upper secondary and higher education. However, some of its limitations include the difficulty of the commands in the input bar, which are not intuitive or easy to handle, particularly for users without prior programming experience (Wassie & Zergaw, 2019). In the same vein, Weinhandl (2020) notes that integrating technological tools such as GeoGebra, along with constructivist educational approaches, has represented an additional workload for some students. According to the author, this extra effort in the learning process could also be leveraged in exams or other forms of formal assessment. However, it is also recognized that this effort brings benefits, such as the development of student skills related to experimentation, visualization, and the identification of mathematical invariants, as a result of students' interaction with the objects represented in the software's graphical environment.

During the literature review conducted, no similar studies were found on the topic addressed in this work. Therefore, one of the main contributions of this documentary study is to provide an overview of the most current research results conducted globally regarding the continuity of real-variable functions. However, it is necessary to acknowledge that this work has some limitations, such as having searched only in three databases, having analyzed a small number of articles, and considering a four-year time period. Based on the above, it would be advisable to conduct a quantitative study and extend the search to include other databases, as well as analyze a larger number of works published over a broader time period. Additionally, empirical research should be carried out to address the learning of continuity in order to understand more about the difficulties students encounter when tackling this topic in the mathematics classroom. Furthermore, workshops or continuing education courses could be organized to train teachers in the use of technology during the teaching-learning process of mathematics in general, and continuity of functions in particular.

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