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Learning obstacle instrument analysis of proportion concept with praxiology framework

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Abstract.
This study explores the development and application of a praxiology-based instrument designed to identify and address learning obstacles in the teaching of the concept of proportion in early algebra. Proportion plays a crucial role in connecting concrete numerical concepts with abstract algebraic thinking; however, many students face significant challenges when transitioning from numerical to algebraic representations. The study utilizes a praxiology framework, which emphasizes the relationship between the type of mathematical task, the techniques employed by students, and the underlying theoretical principles that shape students' understanding. This framework provides a deeper understanding of how task design can influence students' mathematical practices, making it particularly effective for diagnosing learning obstacles in proportion-related tasks. The instrument, which was applied to seventh-grade students in Bandung, Indonesia, consists of five types of tasks aimed at developing students' skills in arithmetic-algebraic representations and solving linear equations in both abstract and contextual forms. By predicting students' potential solutions and analyzing their problem-solving strategies, the study highlights how the praxiological approach facilitates the identification of key difficulties in students' understanding of proportion. The findings demonstrate that this approach not only helps to diagnose learning obstacles more effectively but also supports the creation of targeted instructional strategies that improve students' grasp of proportional reasoning. This research contributes valuable insights into the use of praxiology in mathematics education, offering a robust framework for analyzing and overcoming learning obstacles in early algebra instruction.

Keywords:
Early algebra; learning obstacles; praxiology; proportion; task design

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INTRODUCTION

The concept of proportion is a cornerstone in early algebra and plays a pivotal role in bridging the concrete world of numbers with the abstract realm of algebraic variables. Mastering proportion is essential for developing complex mathematical problem-solving skills, as it not only underpins various areas of arithmetic but also forms the foundation for understanding more advanced concepts such as linear equations and functions (Van de Walle, Karp, & Bay-Williams, 2014). However, students frequently encounter significant challenges when learning about proportion. These challenges often stem from difficulties in transitioning from concrete numerical understanding to the abstract manipulation of algebraic variables, as well as a lack of real-world contextualization and problems with visual representations (Charalambous et al., 2010; Fernández et al., 2024).

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Learning obstacles in proportion are particularly important to address because they can hinder students' ability to apply proportional reasoning, which is vital not only in mathematics but in everyday life situations such as financial planning, scale models, and data interpretation. Without a solid understanding of proportion, students may struggle in later stages of their mathematical education, especially in algebra and beyond. Given the foundational nature of proportion and its central role in developing algebraic thinking, addressing these learning obstacles is crucial for students' long-term mathematical success.

A key approach to overcoming these learning barriers is through praxiology, a framework that examines mathematical practices in context. Praxiology focuses on the relationship between tasks, techniques, and the underlying theoretical principles, offering a comprehensive approach to understanding and analyzing students' mathematical thinking. The praxiological framework has been shown to be particularly useful for identifying learning obstacles, as it allows for a more nuanced understanding of how students engage with mathematical tasks (Trouche, 2016). By analyzing the tasks and techniques students employ, praxiology provides insights into the cognitive and conceptual challenges they face, which can then inform the design of more effective learning tools.

In the context of proportion, praxiology can be used to structure tasks that both diagnose and address students' difficulties. Bosch and Gascón (2006) introduced mathematical praxeology as a model for understanding mathematical knowledge through its practical application. By analyzing tasks, techniques, and the associated theoretical frameworks, praxiology enables a deeper exploration of how students solve problems and the obstacles they encounter. This approach, when applied to the teaching of proportion, allows for the identification of specific learning obstacles, such as misconceptions about ratios or difficulties in applying proportion in diverse contexts.

Preliminary research has indicated that praxiological analysis can significantly enhance the design and effectiveness of instruments aimed at detecting learning obstacles. By applying this framework to the development of a learning obstacle instrument, this study seeks to build on prior work by providing a more detailed understanding of how praxiological analysis can be used to identify and address students' difficulties in learning proportion. This study also builds on the work of Putra and Witri (2017), who demonstrated the efficacy of praxiological tools in mathematics education, and aims to develop an instrument specifically tailored to the challenges of early algebra and proportion.

This research makes three important contributions to the field of mathematics education. First, it designs and tests an instrument based on the praxiology framework to analyze learning obstacles in the teaching of proportion. By aligning the instrument with real and contextualized mathematical practices, this study offers a more accurate method for identifying barriers to understanding. Second, the research provides an in-depth analysis of learning obstacles specific to the concept of proportion, offering new insights into how these difficulties manifest and how they can be overcome. Finally, the study introduces a praxiology-based evaluation framework to assess the effectiveness of the instrument in identifying and addressing learning obstacles. This evaluation framework provides a valuable tool for educators seeking to improve students' understanding of proportion through targeted instructional strategies.

From the perspective of Didactic Anthropology Theory, mathematical praxiology is an essential tool for exploring the cognitive processes involved in solving mathematical problems and understanding the justifications behind different mathematical approaches (Bosch & Gascón, 2006, 2014; Chevallard, 2019). Previous studies have highlighted the usefulness of praxiological analysis in examining mathematical thinking and its application in teaching (Asami-Johansson, 2021; Lundberg & Kilhamn, 2018). This study extends these insights by applying praxiological analysis specifically to the learning of proportion in early algebra. Through this approach, we aim to contribute to the growing body of research on praxiology in mathematics education and provide new strategies for addressing learning obstacles in the classroom.

By examining the intersection of praxiology and proportion, this study aims to enhance the design of educational instruments that can better diagnose and address students' learning obstacles, ultimately improving their understanding of proportion and supporting their transition to more advanced algebraic concepts. This research fills a gap in the existing literature by offering a more comprehensive and contextually grounded approach to analyzing and overcoming learning obstacles in mathematics education.

The primary objectives of this study are: 1) To design and test a praxiology-based instrument that identifies learning obstacles in the teaching of proportion to early algebra students. 2) To analyze the nature and causes of learning obstacles in the concept of proportion, using a praxiology framework. 3) To develop a praxiology-based evaluation framework for assessing the effectiveness of the instrument in overcoming these obstacles and improving students' understanding of proportion. Through these objectives, this study aims to provide a deeper understanding of the role of praxiology in mathematics education, particularly in the context of early algebra, and offer practical tools for educators to address learning difficulties in proportion.

METHOD

This study investigates the development and application of a learning obstacle instrument aimed at identifying and addressing challenges in understanding the concept of proportion in early algebra. The methodology focuses on the design and implementation of the instrument, as well as the theoretical framework underpinning its development. The instrument consists of five mathematical tasks designed to diagnose and address common learning obstacles associated with the concept of proportion. These tasks were implemented in a seventh-grade classroom at a junior high school in Bandung, Indonesia.

Development of the Learning Obstacle Instrument

The instrument was developed based on a praxiology framework, which emphasizes the relationship between tasks, techniques, technological discourse, and the underlying theoretical principles guiding mathematical problem-solving. Each task was carefully designed to reflect key challenges in students' understanding of proportion, particularly focusing on the transition from concrete numerical representations to abstract algebraic variables, contextual understanding, and the use of visual representations. The five tasks selected for this study were chosen to address these specific learning obstacles, drawing on prior research that identified these as common difficulties for students when learning proportion (Charalambous et al., 2010; Fernández et al., 2024).

The selection of the five tasks was informed by both theoretical considerations and pedagogical needs. These tasks aimed to reflect real-life situations where proportion is applicable and to require students to engage in a range of mathematical practices, from basic arithmetic to algebraic reasoning. The tasks were structured to allow for the observation of various mathematical techniques and the identification of the learning obstacles students might encounter.

Sample and Participant Selection

The study was conducted in a seventh-grade classroom at a junior high school in Bandung, Indonesia, comprising 30 students (15 males and 15 females). The selection of this sample was based on the typical curriculum in Indonesian junior high schools, where the concept of proportion is introduced as part of early algebra instruction. This class was chosen because the students were at the appropriate stage in their mathematical education to encounter difficulties related to the concept of proportion.

The sample size of 30 students was determined to provide a sufficient number of participants to identify common patterns in learning obstacles while ensuring practical feasibility within the classroom setting. A sample of this size also allows for meaningful data analysis without overwhelming the research process.

Instrument Validation and Reliability

To ensure the reliability and validity of the instrument, a pilot study was conducted prior to the main implementation. The pilot study involved a smaller group of 10 students who completed the tasks, and their responses were analyzed to identify any ambiguities in the task design or areas where students struggled. Based on feedback from the pilot study, the instrument was refined to improve clarity and alignment with the research objectives.

Additionally, the instrument's validity was assessed through expert review. A panel of mathematics educators with experience in both classroom teaching and educational research reviewed the tasks for their alignment with the theoretical framework and their ability to diagnose the specific learning obstacles associated with the concept of proportion. Adjustments were made based on expert feedback to ensure that the tasks addressed the intended learning outcomes and accurately captured the difficulties students face.

Reliability was assessed through test-retest procedures. The tasks were administered to the same group of students at two different points in time, and the consistency of their responses was evaluated. The results indicated a high level of consistency, suggesting that the instrument was reliable for measuring students' understanding of proportion and identifying learning obstacles.

Data Collection and Analysis

The primary data collection method involved administering the five tasks to the students during regular mathematics lessons. The tasks were designed to assess different aspects of proportional reasoning, including arithmetic-algebraic representations and the application of proportion in contextualized scenarios. Student responses were recorded, and their problem-solving techniques were carefully analyzed to identify common patterns of difficulties and misconceptions.

Data analysis was conducted using a mathematical practices approach, which involves examining the relationships between the tasks, the techniques students employed to solve them, and the theoretical frameworks that informed those techniques. The approach focuses on understanding how students interact with mathematical tasks and how their problem-solving processes reveal underlying learning obstacles. This method allowed for a comprehensive analysis of students' responses, providing insights into the specific challenges they faced and the strategies they used to overcome them.

Additionally, praxiological analysis was used to examine the four components of each mathematical task: the type of task, the techniques used, the technological discourse, and the theoretical principles underpinning those techniques. By analyzing these components, we were able to identify how students' mathematical practices related to the conceptual understanding of proportion and where these practices diverged from the expected solutions.

Limitations of the Methodology

While this methodology provides a comprehensive approach to analyzing learning obstacles in the teaching of proportion, several limitations must be acknowledged. First, the study was conducted in a single classroom, which may limit the generalizability of the findings to other educational contexts. The specific cultural and educational setting in Bandung, Indonesia, may have influenced the way students approached the tasks and their understanding of proportion. Future research should consider replicating this study in different regions or countries to explore whether the identified learning obstacles are universally applicable.

Second, the sample size of 30 students, while sufficient for this study, is relatively small. A larger sample would allow for a more robust analysis of the patterns in student responses and could provide more statistically significant insights into the effectiveness of the instrument. Furthermore, future studies could explore the long-term effectiveness of praxiology-based instruments by tracking students' progress over an extended period.

Finally, while the study employed a praxiological approach to identify learning obstacles, it did not explore the effectiveness of specific instructional interventions that could address these obstacles. Future research could examine how the instrument can be used in conjunction with targeted teaching strategies to improve students' understanding of proportion.

In summary, the methodology outlined in this study provides a detailed and structured approach to developing and implementing a praxiology-based instrument for identifying learning obstacles in the teaching of proportion. By combining theoretical considerations, expert validation, and rigorous data analysis, this study contributes to the growing body of research on the use of praxiology in mathematics education. Despite some limitations, the findings suggest that the instrument is effective in diagnosing learning obstacles and provides valuable insights into the specific challenges students face in mastering the concept of proportion.

RESULTS AND DISCUSSION

In the transposition process to develop teaching objects, the researcher analyzed the mathematics learning process in elementary schools to identify various problems that occur, and analyzed the learning obstacles of the concept of proportionality in secondary schools. The following discusses the learning obstacle analysis of the concept of proportionality in secondary school students based on each of the given problems. The researcher also explained the problems based on praxiology, so that there are techniques, technologies and theories that underlie the task.

Problem 1

The task type in problem 1 is a missing value problem. The familiar context is in food nutrition labeling, the unit rate is not given, and the possibility of being tricked by the absence of integer change factors between the given ratio pairs. The praxeological explanation presented in Table 1 is the subject of discussion in problem 1.

Table 1. Praxiology in problem 1

Task 1	Technique 1	Technology 1	Theory 1
In 8 grams of snack food, there are 48 calories. In 16 grams, there are 96 calories. How many calories are in 20 grams of snack food?	The correct technique can be seen from the student's answer in Figure 4.6, using the unit ratio approach. The unit ratio is determined by simple division across the size space, and then the unit ratio is used as a multiplier to determine the number of calories, with the unit ratio being 6 calories/1 gram, then multiplied by 20 grams to get 120 calories.	An analysis of the multiplication structure that provides an argument for the technique used is as follows. Gram Calories 8 48 16 96 20 ? An integer between the unit ratio size spaces of 6 calories/1 gram is available. The unit rate must be derived. There is no integer in the scalar factor measure space for the computation required for 20 grams.	The theory that justifies the technology is proportionality as a linear relationship between two covariant quantities according to the model y = mx, where m is the unit rate (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In a proportional situation, two invariant unit rates exist across the size space. The unit rates are reciprocal and define inverse functions: y = mx and x = (1/m)y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

Tasks

In 8 grams of snack food, there are 48 calories. In 16 grams there are 96 calories. How many calories are in 20 grams of snack food?

Learning Obstacle

The researchers identified students' learning obstacles related to Problem 1, as illustrated in Figure 1.

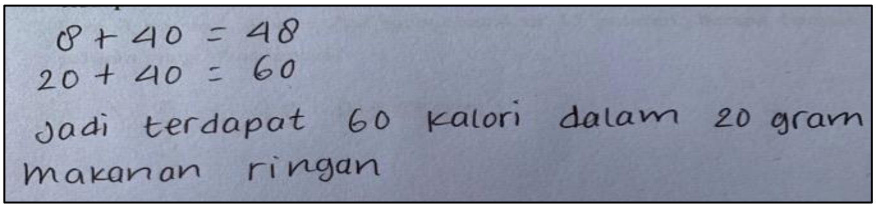

Figure 1. Learning obstacle 1 in problem 1

The first learning obstacle is an incorrect addition strategy based on the first pair of grams of calories. Students work by (Number of grams) + 40 calories. It can be seen in Figure 1, students mistakenly use the addition strategy in the first statement, 8 + 40 = 48, resulting in further errors in the number of calories asked, 20 + 40 = 60. Students skip the second statement of the problem which states there are 16 grams in 96 calories, by not testing it through the method he obtained such as 16 + 40. Students do not use all the statements given, so students arrive at the wrong conclusion.

The second learning obstacle is the erroneous development of the change factor approach. The addition reasoning where calories increased by 48 with the increase in grams doubled, from 8 grams to 16 grams. Then 96 + 48 (48+48+48 or 48 x 3) is for a threefold increase in grams, which is 8 grams x 3 = 24 grams of snacks, not 20 grams as asked. Meanwhile, students worked on 48 + 48 = 96, 96 + 48 = 144 as shown in Figure 2.

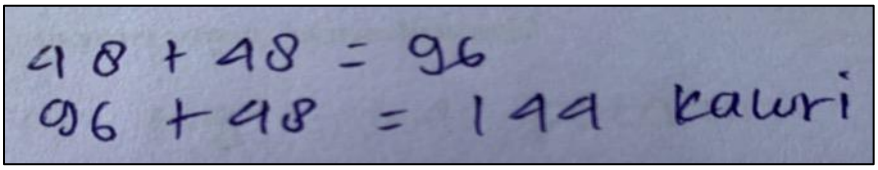

Figure 2. Learning obstacle 2 in problem 1

The third learning obstacle is the development of an erroneous change factor approach. The change factor approach is another approach that students use intuitively in proportional reasoning. This method utilizes the scalar multiplication relationship in measuring space. Multiplying, tripling, and so on are scalar operations. The scalar factor is not a constant, no constant factor is needed. Seen from Figure 3, students write 48 x 2 = 96, 96 x 2 = 192, the number of calories from the number of grams is multiplied by students according to the increase in grams, namely 16/8 = 2, this is not constant for all known grams, students should determine the number of calories by another constant factor, namely 20/8 if using the first description, or 20/16 if using the second description. Students may be fooled by the absence of integer change factors among the given ratio pairs.

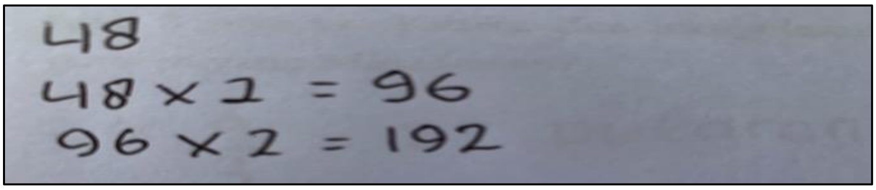

Figure 3. Learning obstacle 3 in problem 1

Technique

The correct technique can be seen from the students' answers in Figure 4, using the unit ratio approach. The unit ratio is determined by simple division across the measurement space, and then the unit ratio is used as a multiplier to determine the number of calories, with the unit ratio being 6 calories/1 gram, then multiplying by 20 grams to get 120 calories.

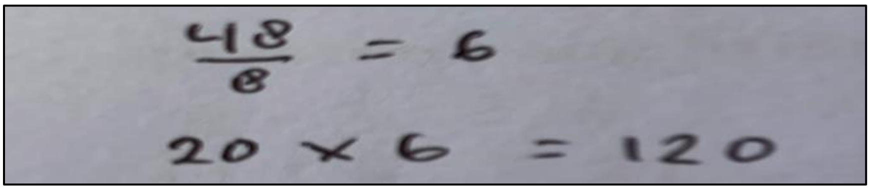

Figure 4. Answer to problem 1

Technology

Analysis of the multiplication structure which provides arguments for the technique used is as follows.

Grams	Calories
8	48
16	96
20	?

Integers between the unit ratio measurement spaces of 6 calories/1 gram are available. The unit rate must be reduced. There are no integers in the scalar factor measurement space for the computation required for 20 grams.

Theory

The theory that justifies this technology is proportionality as a linear relationship between two covariative quantities according to the model y = mx, where m is the unit rate (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In the proportional situation, two invariant unit rates exist over the entire measure space. The unit rate is reciprocal and defines the inverse function: y = mx and x = (1/m) y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

Problem 2

The second type of task (as outlined in Table 2), which is a generalization to the rule $y = mx$ which can be used to solve any pair of rates in a given proportion relative to the situation. The context used is the familiar context of unit rate. Assignments are non-routine tasks and students may not be familiar with the notation that may be used.

Table 2. Praxiology in problem 2

Task 2	Technique 2	Technology 2	Theory 2
In 8 grams of snacks, there are 48 calories. In 16 grams there are 96 calories. What rules can be used to determine the number of calories in each gram of snack food?	The correct technique uses the invariant unit ratio of 6 calories per 1 gram to the relationship structure y = mx, such as y = 6x, so that 48 calories = 6 . 8 grams and 96 calories = 6 . 12 grams. Generalizations were obtained as shown in the students' answers as shown in Figure 4.10, namely (⊡ gram). 6 = (⊡ calories).	Analysis of the multiplication structure which provides arguments for the technique used is as follows. Grams Calories 8 48 16 96 x 6x Integer numbers between unit ratio measuring spaces of 6 calories/1 gram are available. The unit ratio must be reduced.	The theory that justifies this technology, namely, proportionality is a linear relationship between two covarying quantities according to the model y = mx, where m is the unit rate. All corresponding pairs of speeds (x, y) are located on the graph of the line y = mx, which passes through the origin (Karplus et al., 1983; Lamon, 2007; Post et al., 1988).

Task

In 8 grams of snacks, there are 48 calories. In 16 grams there are 96 calories. What rules can be used to determine the number of calories in each gram of snack food?

Learning Obstacles

Researchers have identified student learning obstacles related to problem 2, as presented in the following figures. The first learning obstacle is the generalization of the wrong addition approach to the problem (refer to Figure 5), namely (⊡ grams) + 40 = (⊡ calories).

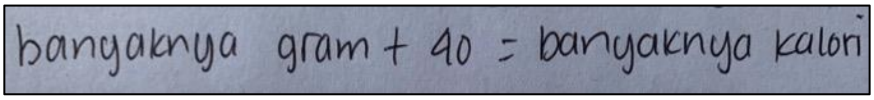

Figure 5. Learning obstacle 1 in problem 2

The second learning obstacle is the incorrect generalization of the addition approach to the problem (Figure 6), namely 48 calories per 8 grams, so (⊡ grams) + 48 = (⊡ calories).

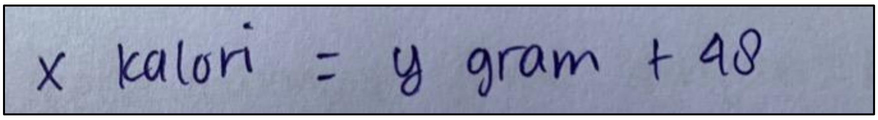

Figure 6. Learning obstacle 2 in problem 2

The third learning obstacle is the incorrect generalization of the whole number change factor approach (refer to Figure 7). The number of calories and the number of grams is multiplied/doubled together, 8 grams x 2 = 16 grams, 48 calories x 2 = 96 calories, so that the generalization is (⊡ grams) x 2 = (⊡ calories).

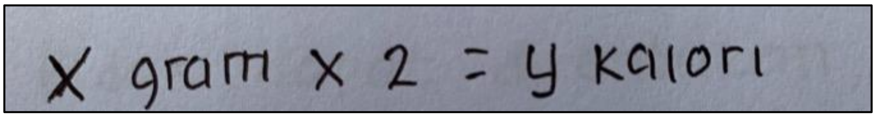

Figure 7. Learning obstacle 3 in problem 2

Technique

The correct technique uses the invariant unit ratio of 6 calories per 1 gram to the relationship structure y = mx, such as y = 6x, so that 48 calories = 6 . 8 grams and 96 calories = 6 . 12 grams. Generalizations were obtained as shown in the students' answers as shown in Figure 8, namely (⊡ gram). 6 = (⊡ calories).

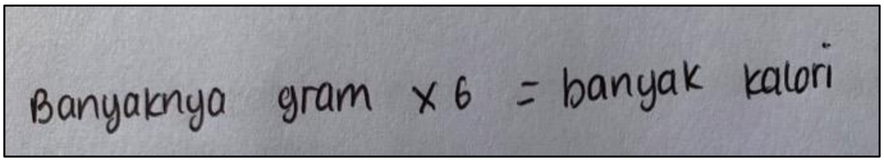

Figure 8. Answer to problem 2

Technology

Analysis of the multiplication structure which provides arguments for the technique used is as follows.

Grams	Calories
8	48
16	96
x	6x

Integer numbers between unit ratio measuring spaces of 6 calories/1 gram are available. The unit ratio must be reduced.

Theory

The theory that justifies this technology, namely, proportionality is a linear relationship between two covarying quantities according to the model y = mx, where m is the unit rate. All corresponding pairs of speeds (x, y) are located on the graph of the line y = mx, which passes through the origin (Karplus et al., 1983; Lamon, 2007; Post et al., 1988).

Problem 3

Task type 3 presented in Table 3 is disproportionately challenging. The context employed is a familiar context, specifically running.

Table 3. Praxiology in problem 3

Task 3	Technique 3	Technology 3	Theory 3
Mina and Zea ran equally fast around a track. Mina started running first. When Mina had run 9 laps, Zea had only run 3 laps. When Zea completed 15 laps, how many laps did Mina complete?	The correct technique is that Mina has run 6 more laps than Zea, so 15 + 6 = 21.	Analysis of the multiplication structure which provides arguments for the technique used is as follows. Mina Zea 3 +6 9 15 +6 ? If the situation is proportional, there is a unit ratio and a whole number change factor. This is a quantitative distraction that can lead to the erroneous use of proportional reasoning in non-proportional tasks.	The theory that justifies this technology, namely, proportionality is a linear relationship between two covarying quantities according to the model y = mx, where m is the unit rate. All corresponding pairs of speeds (x, y) are located on the graph of the line y = mx, which passes through the origin (Karplus et al., 1983; Lamon, 2007; Post et al., 1988).

Task

Mina and Zea ran equally fast around a track. Mina started running first. When Mina had run 9 laps, Zea had only run 3 laps. When Zea completed 15 laps, how many laps did Mina complete?

Learning Obstacles

Researchers identify students' learning obstacles related to problem 3. The first learning obstacle is the use of proportional reasoning in non-proportional situations, as in the student's answer shown in Figure 9, namely 15 x (9/3) = 45.

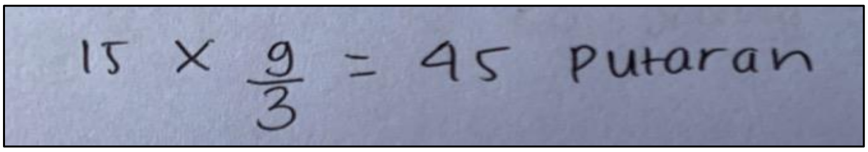

Figure 9. Learning obstacle 1 in problem 3

The second learning obstacle is the incorrect use of proportional reasoning into a non-proportional situation, as in the student's answer shown in Figure 10, which is 9 x (9/3) = 27.

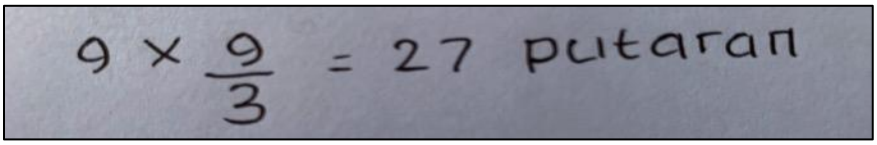

Figure 10. Learning obstacle 2 in problem 3

The third learning obstacle is incorrect additive reasoning, Mina has run 9 laps more than Zea, so 15 + 9 = 24 (Figure 11).

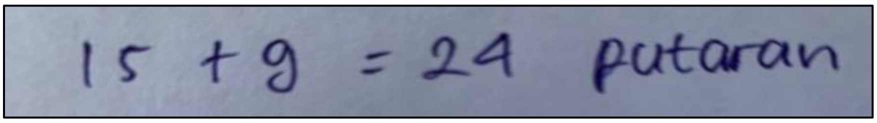

Figure 11. Learning obstacle 3 in problem 3

Technique

The correct technique is that Mina has completed six additional laps compared to Zea, resulting in a total of 15 + 6 = 21 laps. This is illustrated in Figure 12.

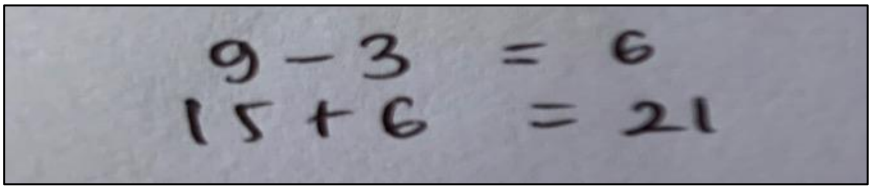

Figure 12. Answers to problem 3

Technology

The analysis of the multiplication structure that provides an argument for the technique used is as follows.

Mina Zea
3 +6 9
15 +6 ?

If the situation is proportional, there are unit ratios and integer change factors. This is a quantitative distraction that can bring erroneous use of proportional reasoning into non-proportional tasks.

Theory

The theory that justifies the technology, namely, proportionality is a linear relationship between two quantities that covary according to the model y = mx, where m is the unit rate. All corresponding rate pairs (x,y) lie on the graph of the line y = mx, which passes through the origin (Karplus et al., 1983; Lamon, 2007; Post et al., 1988).

Problem 4

Task type 4 is a non-routine task (as indicated in Table 4). Ignorance with notation is possible. The previous problem leads students to think about using scalar factors of change in rules. Students should be able to cope with this complexity.

Table 4. Praxiology in problem 4

Task 4	Technique 4	Technology 4	Theory 4
Diana is ordering pizza for a birthday party. She estimates that 3 pizzas will be enough for 10 people. What is the rule that can be used to determine the number of pizzas Diana should buy for a certain number of people?	The technique used is the use of an invariant unit ratio, namely 3/10, so that (# Pizza) = (3 pizzas per 10 people) x (# People).	There are no integers for the two invariant unit ratios 10/3 and 3/10. The unit ratios should be determined according to the context so as to solve the problem between the size spaces.	The theory justifying the technology is that proportionality is a linear relationship between two covariant quantities according to the model y = mx, where m is the unit rate. (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In a proportional situation, two invariant unit rates exist across the size space. The unit rates are reciprocal and define inverse functions: y = mx and x = (1/m)y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

Task

Diana is ordering pizza for a birthday party. She estimates that 3 pizzas will be enough for 10 people. What is the rule that can be used to determine the number of pizzas Diana should buy for a given number of people?

Learning Obstacle

The researcher identified students' *learning obstacles* related to problem 4. The first learning obstacle is the use of the wrong unit ratio (refer to Figure 13 and Figure 14). Students use the information, 3 pizzas per person as the unit ratio between the given size spaces, which should be 3 pizzas per 10 people.

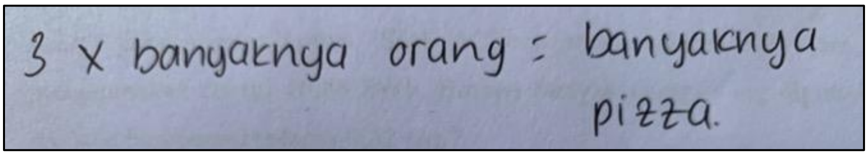

Figure 13. Learning obstacle 1 in problem 4

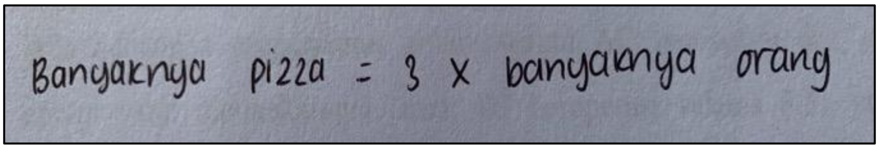

Figure 14. Learning obstacle 1 in problem 4

The second learning obstacle is the use of other unit ratios that have different contexts of use, as depicted in Figure 15. Students utilize the opposite of the unit ratio that should not be used in this rule.

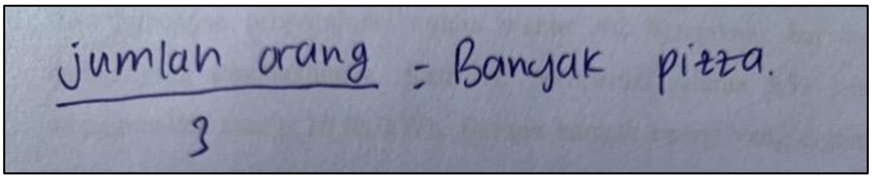

Figure 15. Learning obstacle 2 in problem 4

The third learning obstacle is the use of change factors that are different in context. The scalar factor of change from the previous task is used instead of the invariant unit rate, as depicted in Figure 16.

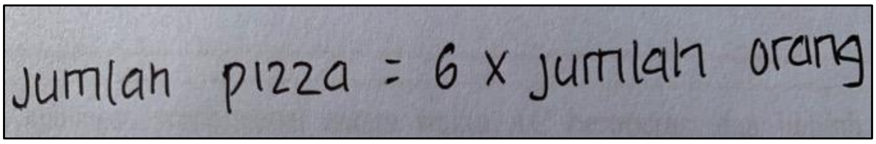

Figure 16. Learning obstacle 3 in problem 4

Technique

The technique used is the use of the invariant unit ratio, which is 3/10, so that (# Pizza) = (3 pizzas per 10 people) x (# People).

Technology

There are no integers for the two invariant unit ratios 10/3 and 3/10. The unit ratio should be determined according to the context so as to solve the problem between the size spaces.

Theory

The theory that justifies the technology, namely, proportionality is a linear relationship between two covariant quantities according to the model y = mx, where m is the unit rate. (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In a proportional situation, two invariant unit rates exist across the size space. The unit rates are reciprocal and define inverse functions: y = mx and x = (1/m)y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

Problem 5

The task type in problem 5 is a missing value problem (refer to Table 5). The scientific context is the energy used by the air conditioner to operate, the unit rate is not given, and there is no integer change factor between the given ratio pairs.

Table 5. Praxiology in problem 5

Task 5	Technique 5	Technology 5	Theory 5
There is a proportional relationship between the time the air conditioner operates and the amount of energy it uses. When the air conditioner operates for 3.51 hours, it uses 10.88 kWh of energy. How much energy does the air conditioner use when it operates for 4.62 hours?	The correct technique is shown in Figure 4.34, (kWh of energy) = (10.88 kWh/3.51 hours) x (4.62 hours) = 14.32 kWh.	An analysis of the multiplication structure that provides an argument for the technique used is as follows. Hour kWh 3.51 10.88 x(10.88/3.51) 4.62 14.32 x(10.88/3.51) Use of unit rate approach in solving proportional problems.	The theory justifying the technology is that proportionality is a linear relationship between two covariant quantities according to the model y = mx, where m is the unit rate. (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In a proportional situation, two invariant unit rates exist across the size space. The unit rates are reciprocal and define inverse functions: y = mx and x = (1/m)y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

Task

There is a proportional relationship between the time an air conditioner operates and the amount of energy it uses. When an air conditioner operates for 3.51 hours, it uses 10.88 kWh of energy. How much energy does the air conditioner use when it operates for 4.62 hours?

Learning Obstacle

The researcher identified students' *learning obstacles* related to problem 5. The first learning obstacle is the wrong unit rate approach. As shown in Figure 17, the student answered (kWh of energy) = (10.88 kWh/4.62 hours) x (3.51 hours) = 8.27 kWh.

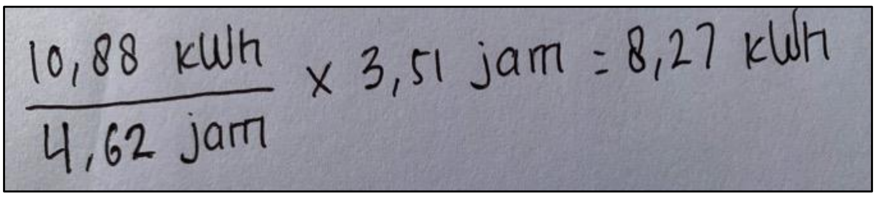

Figure 17. Learning obstacle 1 in problem 5

The second learning obstacle is the wrong additive strategy. The difference in hours, 4.62 - 3.51 = 1.11 hours, was added to the kWh of energy used when the AC ran for 3.51 hours. Thus, 10.88 + 1.11 = 11.99 (Figure 18).

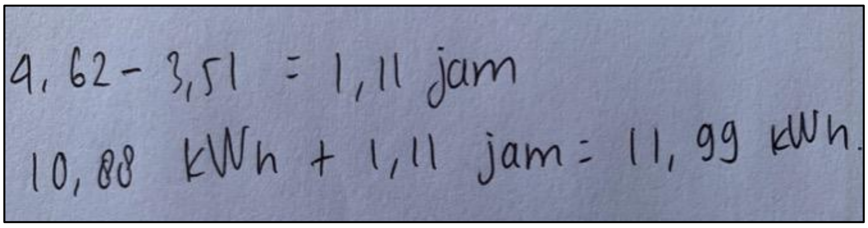

Figure 18. Learning obstacle 2 in problem 5

The third learning obstacle is that students do not reason quantitatively. The student answered that it could not be determined (Figure 19). The quantitative element (decimal number) given in this task makes it a problem that students cannot solve.

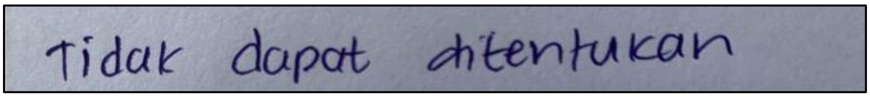

Figure 19. Learning obstacle 3 in problem 5

Techniques

The right technique looks like the student's answer in Figure 20, (kWh of energy) = (10.88 kWh/3.51 hours) x (4.62 hours) = 14.32 kWh.

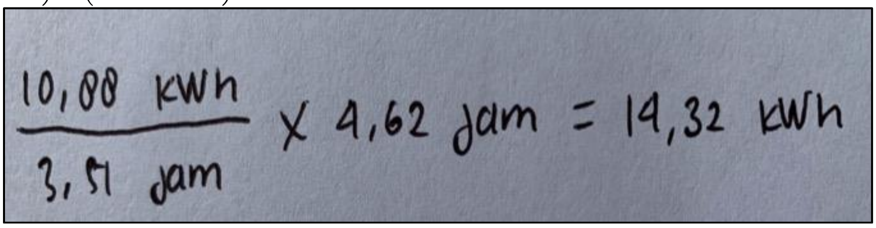

Figure 20. Answers to problem 5

Technology

The analysis of the multiplication structure that provides argumentation for the technique used is as follows.

Hour	kWh
3.51 x(10.88/3.51)	10.88
4.62 x(10.88/3.51)	14.32

Use of unit rate approach in solving proportional problems.

Theory

The theory that justifies the technology, namely, proportionality is a linear relationship between two covariant quantities according to the model y = mx, where m is the unit rate. (Karplus et al., 1983; Lamon, 2007; Post et al., 1988). In a proportional situation, two invariant unit rates exist across the size space. The unit rates are reciprocal and define inverse functions: y = mx and x = (1/m)y (Lamon, 2007; Post et al., 1988, Vergnaud, 1983).

This study demonstrates that the learning obstacle instrument, grounded in mathematical praxiology, plays a crucial role in facilitating the construction of mathematical knowledge, especially in relation to the concepts of proportion, variables, algebraic representations, operations, and linear equations. As articulated in epistemology, knowledge is typically understood as justified true belief (Audi, 2010), which implies that for a belief to be considered knowledge, it must be both true and justified. The process of constructing mathematical knowledge involves not only identifying what is true but also how beliefs about mathematical concepts are formed, justified, and validated. The findings of this study align with this perspective by highlighting the importance of multiple sources of justification, such as perceptual observation, memorial analysis, and introspective awareness, in the development of justified beliefs about mathematical concepts (Audi, 2010; Chisholm et al., 1989).

In the context of praxiology, mathematical tasks do not merely represent abstract concepts but also serve as practical tools for engaging students in the epistemic process of justification and knowledge construction. By analyzing the tasks and student responses within the learning obstacle instrument, this study reveals how students develop their mathematical understanding through active engagement with tasks that require them to formulate, validate, and refine their solutions. This aligns with the praxiological view that knowledge is constructed through action and reflection, emphasizing the role of practical engagement in learning (Stein et al., 2008; Rittle-Johnson et al., 2019).

Connection to Praxiological Theory

The praxiological theory underscores the importance of the relationship between practice and theory in knowledge construction. In the context of this study, the tasks embedded within the learning obstacle instrument provide a framework through which students can engage in mathematical actions—solving problems, making predictions, and formulating new ideas. These tasks are not simply exercises in computation but opportunities for students to develop their own mathematical reasoning, a process that aligns closely with the praxiological concept of knowledge as something actively constructed by the learner (Baker, 2019; Swanson, 2019).

The instrument's design—incorporating tasks related to proportion, algebraic representations, and linear equations—provides a structure within which students can confront and overcome learning obstacles. For example, by engaging with tasks that require them to justify their reasoning through perceptual, memorial, and introspective awareness, students are encouraged to actively construct new mathematical knowledge. This mirrors the praxiological framework's emphasis on student independence and cognitive engagement in learning (Yeager et al., 2020). By using the instrument to predict and analyze students' responses, teachers can identify where students may face difficulties, which enables them to provide targeted support and facilitate the development of more robust mathematical justifications.

Supporting and Extending Existing Research

The findings of this study also build on existing research regarding the role of prediction and anticipation in teaching practices. As noted by Vale et al. (2019), predicting student responses allows teachers to anticipate difficulties and adjust their instructional strategies accordingly. This study reinforces that prediction is a key component of effective teaching, particularly in mathematics, where the complexity of tasks often challenges students' conceptual understanding. Moreover, the results are consistent with studies by Lewis et al. (2019) and Llinares et al. (2016), which emphasize that prediction helps teachers understand how students are likely to engage with tasks and which concepts they may struggle with. By incorporating predictions into lesson planning, teachers can identify potential misconceptions early, a practice that not only improves the quality of classroom discussions but also supports the refinement of student thinking (Yilmaz et al., 2019).

The praxiological approach to task design and anticipation of student responses, as demonstrated in this study, provides teachers with the tools to anticipate learning obstacles more effectively. For instance, by understanding the likely trajectories of student thinking, teachers can select and present solutions in a way that facilitates both individual student reflection and collaborative peer discussions. These practices help to create an environment where students can compare and validate their own thinking, improving both their problem-solving strategies and their understanding of mathematical concepts (Chapin et al., 2009).

This study contributes to the growing body of literature on mathematical praxiology and its application in addressing learning obstacles. The findings underscore the value of using the learning obstacle instrument to facilitate the construction of mathematical knowledge through task design, student prediction, and anticipation of responses. By embedding praxiological principles into teaching practices, educators can create more effective learning environments that promote student independence and deeper conceptual understanding. As this study demonstrates, praxiology offers a robust framework for understanding and addressing learning obstacles in mathematics, with practical implications for teaching, teacher training, and educational policy. Future research should continue to explore the broader applications of this framework and its impact on student learning outcomes across various mathematical domains.

CONCLUSIONS

This study highlights the effectiveness of the learning obstacle instrument in facilitating students' construction of mathematical knowledge, particularly in relation to the concept of proportion and its connections to variables, algebraic representations, operations, and linear equations. By integrating didactic anthropology theory, the study demonstrates how the epistemic nature of mathematical tasks can support students in independently using their perceptual and memorial potential. Specifically, the instrument allows students to engage with mathematical tasks in ways that encourage the formulation, action, and validation of mathematical ideas, fostering both conceptual understanding and independent problem-solving abilities.

From the perspective of didactic anthropology, the instrument provides an opportunity for students to move beyond mere procedural learning by engaging in mental actions and modes of thinking that lead to new mathematical formulations. This process not only helps students internalize mathematical concepts but also empowers them to construct their own mathematical objects. The findings suggest that this kind of active, self-directed learning can enhance students' mathematical thinking and provide a deeper understanding of the relationships between different mathematical concepts.

For educators, the study offers several practical implications. First, incorporating learning obstacle instruments into the classroom can provide valuable insights into student thinking, particularly in identifying misconceptions or gaps in understanding. Teachers are encouraged to use these tools to anticipate students' solutions to mathematical tasks, which can help in tailoring instruction and providing more targeted interventions. In particular, teachers should focus on creating opportunities for students to independently explore mathematical concepts, as this process fosters deeper conceptual understanding and promotes critical thinking.

Moreover, it is recommended that teachers incorporate tasks that encourage students to engage with the epistemic nature of mathematical problems—tasks that require students to formulate, test, and validate their solutions. This approach can be particularly effective in teaching complex concepts such as proportion, algebraic reasoning, and linear equations. By allowing students to work through tasks that involve both cognitive and procedural challenges, teachers can support the development of independent mathematical reasoning.

For educational policymakers, the findings suggest the importance of integrating tools like the learning obstacle instrument into teacher professional development programs. Training educators to effectively use such instruments can help them better understand and address learning barriers in mathematics. Policymakers should also consider fostering collaboration between educators, researchers, and curriculum designers to develop resources that align with the praxiological approach and support student-centered learning.

While this study provides important insights, there are several avenues for future research that could further elucidate the findings. Future studies could explore how the learning obstacle instrument impacts students' long-term retention and mastery of mathematical concepts, particularly in complex areas such as algebra and proportional reasoning. Additionally, research could investigate how the instrument can be adapted for use in diverse educational contexts, including different cultural or educational settings, to determine its broader applicability and effectiveness.

Further investigation into the role of the teacher in facilitating the epistemic learning process is also needed. Specifically, research could explore how teachers can best scaffold students' independence in formulating and validating mathematical concepts, and what instructional strategies are most effective in supporting this process. Additionally, examining the impact of technological tools on students' use of the learning obstacle instrument could provide valuable insights into how digital resources might enhance or complement traditional teaching methods.

In conclusion, this study underscores the potential of the learning obstacle instrument to support the construction of mathematical knowledge, particularly in fostering student independence and critical thinking. By incorporating these findings into educational practices, educators can create more effective learning environments that empower students to engage with mathematics at a deeper level. Further research in this area will be critical in refining these approaches and expanding their applicability to a broader range of educational contexts.

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