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Analyzing conceptual and procedural errors in solving routine problems on gradients and straight-line equation among prospective mathematics teachers

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Abstract.
This study examines the types of errors made by prospective mathematics teachers in solving routine problems involving gradients and linear equations, which are fundamental to Analytic Geometry. Employing a qualitative descriptive methodology, the research involved 35 mathematics education students who completed a semi-structured written test designed to assess both conceptual comprehension and procedural proficiency. Eleven responses containing incomplete or incorrect solutions were purposively selected for detailed analysis. Data were analyzed using Miles and Huberman's interactive model, encompassing stages of data reduction, presentation, and conclusion derivation. Errors were systematically categorized into conceptual, procedural, and other types, with further refinement into 18 specific misconception subtypes. The findings revealed persistent misconceptions in gradient interpretation, improper application of linear equation forms, and frequent procedural errors in algebraic simplification and negative sign operations. Several students exhibited overlapping error types, indicating a strong correlation between conceptual gaps and procedural breakdowns. These results emphasize the significance of designing instructional interventions that simultaneously enhance conceptual and procedural proficiency.

Keywords:
Analytic geometry; conceptual and procedural errors; error analysis; mathematics education; prospective mathematics teacher

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INTRODUCTION

Analytic geometry serves as a bridge between algebra and geometry, facilitating students' ability to establish a direct connection between visual and symbolic representations (Halidin, 2022). A fundamental concept in its instruction is the gradient and straight-line equation, which is pivotal for comprehending advanced concepts such as tangent equations to conic sections (Busrah & Buhaerah, 2021; Susilo & Hariyani, 2019). The understanding of tangents to circles, parabolas, ellipses, and hyperbolas hinges on the comprehension of gradients and straight-line equations, as their derivation relies on slope-algebraic relationships (Sukirman, 2016). For instance, tangent equations for circles necessitate the computation of perpendicular gradients, while parabolas and ellipses necessitate algebraic manipulation of linear expressions (Yunita, 2017).

A comprehensive understanding of gradients and straight-line equations is not merely a prerequisite but a fundamental conceptual foundation that underpins the entire process of constructing and interpreting tangent equations. Without a solid grasp of these concepts, students encounter difficulties in deriving, comprehending, and accurately applying tangent line formulas in analytic geometry, which substantially impedes their progress in advanced topics (Polya, 2014).

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Proficiency in these fundamental concepts frequently manifests in the capacity to resolve routine problems—tasks that can be solved employing conventional strategies and established formulas (Harahap, 2022). Although routine problems may appear simple, effectively resolving them showcases conceptual comprehension, the capacity to discern pertinent information, and proficiency in executing procedures (Saygili, 2017). Conversely, the inability to resolve such issues indicates deficiencies in both conceptual and algorithmic knowledge, which are prerequisites for advancing to more complex thinking tasks.

Despite these findings, various studies have demonstrated that students persistently encounter difficulties in comprehending and resolving problems associated with gradients and straight-line equations. Wahyuni and Alfiana (2022) identified that students' learning outcomes were below average, suggesting a deficiency in their mathematical problem-solving abilities within this subject area. Similarly, Setyaningsih and Firmansyah (2022) reported generally inadequate problem-solving skills among students. Common errors include misinterpretation of gradient formulas, incorrect equation transposition, and arithmetic negligence (Dahlia et al., 2024). These challenges are consistent with Sehajun and Tambunan's (2021) findings, which identified difficulties in daily mathematics assessments—students were unable to solve linear equation problems due to language barriers, conceptual gaps, and computational errors.

A deeper analysis of the cognitive challenges faced by Ismail and Ismail (2024) reveals conceptual difficulties, such as the inability to recall or deduce concepts and incomplete formula writing, as well as algorithmic shortcomings, including poor planning and execution that often result in unanswered questions. These findings collectively highlight systemic gaps in mastering foundational topics, which impede progress toward advanced analytic geometry concepts.

Despite extensive research on students' difficulties with gradients and straight-line equations, critical gaps persist in the literature. Firstly, only a limited number of studies explicitly focus on prospective mathematics teachers' understanding of these concepts, leaving a critical blind spot in teacher education research. For instance, Yuwono and Widyawati (2022) conducted a qualitative case study of mathematics education students and reported that some prospective teachers failed to recognize that equations such as $x = 0$ and $y = 0$ represent straight lines, often misclassifying them as points. This highlights a concerning lack of conceptual clarity at the very foundation of analytic geometry.

The prevailing error classifications often oversimplify learner difficulties by categorizing them broadly into conceptual or procedural domains, neglecting to provide a detailed analysis of specific misconceptions. For instance, Sehole et al. (2023) identified that students frequently misinterpret the y-intercept, commonly assuming that all linear graphs must pass through the origin. This error is often overlooked in conventional assessments. This oversight highlights the necessity for more comprehensive error taxonomies that specifically identify misapplications of the slope formula or algebraic transposition errors, thereby enhancing the effectiveness of instructional interventions.

Consequently, the practical imperative of addressing these deficiencies has been inadequately underscored. A profound conceptual and procedural comprehension among prospective educators is paramount, as their competence directly influences the efficacy of their identification and remediation of analogous misconceptions in their students.

In light of the preceding context, this study aims to elucidate the types of errors incurred by prospective mathematics educators in resolving routine problems pertaining to gradients and linear equations. This research will employ a rigorous analytical framework to systematically categorize errors that transcend the conventional conceptual-procedural dichotomy by identifying specific misconceptions subtypes (e.g., erroneous interpretation of slope, errors in rearranging equations, sign operation errors).

The ensuing error framework will guide the development of precision-targeted instructional materials that address both subject matter knowledge and pedagogical content knowledge deficiencies. These modules will incorporate: (a) misconception-specific remediation activities, (b) error anticipation training, and (c) diagnostic assessment tools for classroom utilization. By converting research findings into practical teacher training resources, this study will directly enhance the quality of mathematics teacher preparation, ultimately resulting in a cascading improvement in how future educators impart these fundamental concepts and address student challenges within their own classrooms.

METHOD

This qualitative descriptive research study aims to identify students' error patterns when solving routine problems involving gradients and straight-line equations. As outlined by Abdurrahman and Moleong (as cited in Fadilah & Bernard, 2021), qualitative descriptive research is an investigation that precisely depicts specific conditions or phenomena, generating qualitative data in the form of verbal or written statements from research subjects.

To gather data, participants were administered a written assessment comprising three routine problems intended to evaluate both conceptual comprehension and procedural proficiency. The test items were formulated based on a comprehensive review of pertinent curricula and prior research on students' challenges. To ensure validity, the items underwent evaluation by two experts in the field of mathematics education. The test items are presented in Figure 1.

Routine Problems

P: (-2, 5)
Q: (-6, -3)

a) Find the gradient of the line PQ.
b) Find the equation of the line PQ in the form $ax + by + c = 0$.
c) Find the equation of the line parallel to PQ that passes through the point R with coordinates $(1, -2)$.

Figure 1. Routine test items on gradients and straight-line equation

Selection Process

The objective of this research is to examine the students' written work on problems related to gradients and straight-line equations, which are fundamental concepts in analytic geometry. The research subjects were 35 undergraduate students enrolled in the Mathematics Education program at Universitas Singaperbangsa Karawang. These students had received a lecture on the topic in a single instructional session and were selected purposively as prospective mathematics teachers whose conceptual understanding and procedural fluency are essential for their future professional roles.

Following the lecture, students completed a semi-closed written test, allowing them to bring personal notes containing relevant formulas. This approach minimized the influence of mere memorization difficulties and emphasized their ability to apply formulas meaningfully to solve routine problems. After correction, 11 of the 35-student works were identified as "imperfect" or "incorrect," indicating substantive errors in reasoning, conceptual application, or procedural execution.

The decision to focus exclusively on these 11 works was methodologically intentional. Since the research aimed to analyze types of errors rather than correctness, selecting erroneous responses permitted a deeper exploration of misconceptions and difficulties that would otherwise remain concealed if only correct solutions were considered. Conversely, the remaining 24 works that were deemed correct still hold relevance in the broader dataset, but they did not provide sufficient material for detailed error analysis. Instead, they served as a comparative backdrop, confirming that the test items were indeed solvable within the specified constraints and that the observed errors were not attributable to ambiguity in the instrument itself.

Data Analysis

The data analysis was conducted in accordance with Miles et al. (2014) framework, which comprises four distinct stages: data collection, data reduction, data visualization, and conclusion derivation (Miles et al., 2014). The primary data source for analysis was the work of 11 imperfect or incorrect students. The analysis categorized errors into two main types: conceptual and procedural errors, as these routine problems required both conceptual comprehension and procedural proficiency (Harahap, 2022). Conceptual errors arise from misunderstandings of mathematical concepts and relationships, manifesting in the following ways: (1) Misinterpretation of problem requirements, (2) incorrect formula selection, and (3) improper formula application (Novitasari & Fitriani, 2021; Rezkia et al., 2025; Sudjanta et al., 2024). Procedural errors occurred due to the incorrect execution of solution steps, including: (a) the misapplication of mathematical rules, (b) the failure to simplify expressions, or (c) the incomplete problem-solving sequences (Lingga et al., 2024; Stovner & Klette, 2022; Susilawati et al., 2024; Wati & Darmawan, 2024). The conclusion-drawing phase specifically identified common misconceptions (e.g., incorrect applications of the gradient formula or algebraic transposition errors) to guide targeted interventions. The general workflow of this study is illustrated in Figure 2.

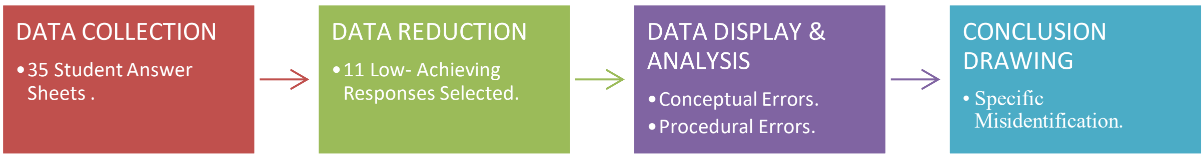

Figure 2. Research flow

Table 1 presents the classification criteria for misconception subtypes (conceptual and procedural) identified in students' solutions to routine gradient and straight-line problems (Sukirman, 2016).

Table 1. Criteria for conceptual and procedural abilities

No	Main Principal	Subtypes Misconception	Terminology
1	Conceptual	1. Gradient when two points are known.	C1
		2. Gradient of parallel lines.	C2
		3. Equation of a line passing through two points.	C3
		4. Equation of a line passing through a given point with a specific gradient.	C4
2	Procedural	1. Subtraction of a negative number by a negative number.	P1
		2. Subtraction of a negative number by a positive number, or vice versa.	P2
		3. Subtraction of a positive number by a positive number.	P3
		4. Division or multiplication of a negative number by a negative number.	P4
		5. Addition of a negative number by a positive number, or vice versa.	P5
		6. Cross multiplication.	P6
		7. Algebraic expansion of the form a(b+c).	P7
		8. Multiplication of a number by a variable.	P8
		9. Multiplication of a positive number by a positive number.	P9
		10. Multiplication of a negative number by a negative number.	P10

No	Main Principal	Subtypes Misconception	Terminology
		11. Multiplication of a positive number by a negative number, or vice versa.	P11
		12. Simplifying an equation through addition or subtraction of a certain value.	P12
		13. Simplifying an equation through multiplication or division by a certain value.	P13
		14. Division or multiplication of a negative number by a positive number, or vice versa.	P14
3	Others	1. Question misinterpretation.	O1
		2. Non-Attempt	O2

Beyond conceptual and procedural elements, Table 1 also documents other potential outcomes observed in students' approaches to routine problems, categorized as 'Main Principal Others' with two distinct misconception subtypes.

RESULTS AND DISCUSSION

Following data reduction, eleven students were identified who provided solutions that were either imperfect or erroneous to the routine problems. A comprehensive analysis of these cases uncovered both conceptual and procedural errors, as detailed in Table 2.

Table 2. Error subtypes distribution

Student	Routine Problem		
	a	b	c
1	O1	P12	C2
2	O1	P6, P12, P13	P13
3	C1, P13, P6	No error	P13
4	No error	P13	O1
5	P4	C3	C2, C4
6	No error	C3	C2, C4
7	P5, P14	C3, P1	P13
8	P14	P12	P7, C2, C4
9	No error	No error	C2
10	C1	C3	O2
11	No error	No error	O1

The data presented in Table 2 reveal a diverse range of conceptual (C), procedural (P), and other (O) errors encountered by the students. For instance, Student 1 exhibited an interpretation error (O1) in problem (a), a simplification error (P12) in problem (b), and a misconception regarding parallel lines (C2) in problem (c). Similarly, Student 5 demonstrated a combination of procedural (P4) and conceptual errors (C3, C2, C4) across the three problem categories. In contrast, some students, such as Student 9, demonstrated accuracy in the initial two categories but committed a conceptual error (C2) in the final category. Student 11, on the other hand, exhibited correctness in the categories (a) and (b) but refrained from attempting problem (c), which was coded as O1. Overall, the results from Table 2 can be summarized and presented in Table 3.

As per Table 3, conceptual errors were encountered a total of 12 times, with the highest concentration observed in misconceptions regarding the gradient of parallel lines (C2, 5 cases) and errors in determining line equations (C3 and C4, 5 cases collectively). Procedural errors were the most prevalent, occurring 15 times in total, particularly involving simplification processes (P12 and P13, 7 cases) and fundamental arithmetic operations involving negative numbers (P4, P5, and P14, 4 cases). Other errors, such as question misinterpretation (O1, 3 cases) and non-attempts (O2, 1 case), were less frequent but still noteworthy. In contrast, seven correct responses (no errors) were identified across the three problems. Overall, the table indicates that procedural errors predominated in students' work, followed by conceptual errors, while problem (c) was the most challenging task, resulting in the highest number of errors.

Table 3. Error subtypes tabulation

Error Code	Problem a	Problem b	Problem c	Frequency	Total
C1	1	–	1	2	12
C2	–	–	5	5	
C3	–	2	1	3	
C4	–	–	2	2	
P1	–	1	–	1	15
P4	1	–	–	1	
P5	1	–	–	1	
P6	1	1	–	2	
P7	–	–	1	1	
P12	–	2	1	3	
P13	1	2	1	4	
P14	2	–	–	2	
O1	2	–	1	3	4
O2	–	–	1	1	
No error	3	3	1	7	7

The findings indicate that students' difficulties in solving routine problems on gradients and straight-line equations can be categorized into two major types of errors: conceptual and procedural. Let us begin with the conceptual error.

Conceptual error reflects students' insufficient understanding of fundamental concepts. The most frequent misconception concerned the gradient of parallel lines (C2), where students often failed to apply the principle that parallel lines share the same slope. Refer to Figure 3.

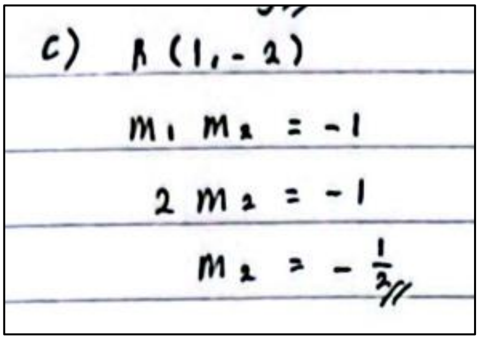

Figure 3. Student misconception in gradient parallel lines concept

This suggests a lack of understanding between slope properties and line equations, which are essential for constructing accurate representations in analytic geometry. Furthermore, misconceptions were observed in determining equations of lines (C3 and C4), indicating that students encountered difficulties in integrating known points or gradients into algebraic expressions. Refer to Figure 4 below for further clarification.

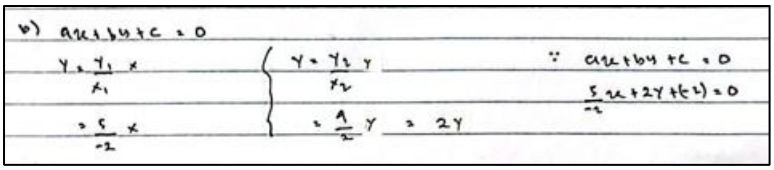

Figure 4. Student misconception in determining equations of line

The errors highlight gaps not only in recalling correct formulas but also in comprehending their geometric significance, consistent with prior research that underscores the pivotal role of conceptual proficiency in analytic geometry (Sehole et al., 2023; Yuwono & Widyawati, 2022). Without this foundation, students are hindered from progressing toward advanced applications, such as deriving tangent equations for conic sections.

Procedural errors were more prevalent overall, indicating deficiencies in algorithmic proficiency despite the availability of formulas. The most prevalent subtypes included simplification errors (P12 and P13), where students failed to apply addition, subtraction, multiplication, or division systematically when manipulating equations. Refer to Figure 5 for further illustration.

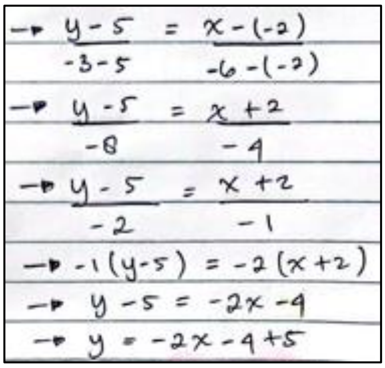

Figure 5. Failed in simplification

Observe Figure 5, particularly the second and third lines from the bottom. A simplification error has occurred. The expression $-1(y-5)$ should be simplified as $-y+5$, not $y-5$. Furthermore, errors in basic arithmetic involving negative numbers (P4, P5, and P14) further demonstrate difficulties at the operational level, which disrupted otherwise correct problem-solving plans. Refer to Figure 6 below for further clarification.

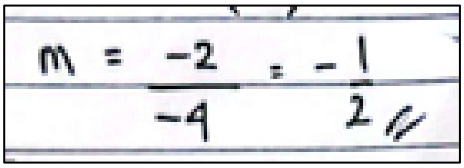

Figure 6. Failed in basic arithmetic with negative numbers

These procedural breakdowns indicate that certain students possessed the ability to recall pertinent formulas but were unable to execute them accurately, suggesting a discrepancy between conceptual comprehension and procedural proficiency. Such errors align with previous reports that routine algebraic manipulation continues to pose a persistent challenge even for prospective educators (Lingga et al., 2024; Stovner & Klette, 2022).

CONCLUSIONS

This study examined the types of errors made by prospective mathematics educators when solving routine problems involving gradients and linear equations. The findings revealed a diverse range of conceptual and procedural errors, as well as other types such as misinterpretation of problems and non-attempts. Conceptual errors included misunderstandings of gradient calculations, incorrect interpretations of parallelism, and difficulties in constructing linear equations. Procedural errors, particularly simplification errors and mismanagement of negative number operations, were most prevalent and indicated a lack of fundamental algorithmic proficiency. Overall, the results demonstrate that while procedural errors are more frequent, conceptual errors are equally critical because they indicate deeper misunderstandings that may impede progress to advanced analytic geometry topics. These findings underscore the significance of designing instructional interventions that simultaneously enhance conceptual comprehension and procedural fluency.

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