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Mathematical multiple representation and mathematical problem-solving abilities of junior high school students employing an open-ended strategy

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Abstract.
This study investigates the effect of the open-ended learning strategy on students' multiple mathematical representation and problem-solving abilities in junior high school mathematics. A quasi-experimental method employing a pretest–posttest control group design was implemented with two classes of eighth-grade students at SMP Negeri 28 Medan. The experimental group received open-ended learning, while the control group received direct instruction. Data were collected through tests of multiple mathematical representation and problem-solving ability, complemented by classroom observations. Quantitative data were analyzed using descriptive statistics, the Mann–Whitney U test, the independent samples *t*-test, and Spearman's rank correlation.The results revealed that students taught using the open-ended learning strategy achieved higher posttest scores in both multiple representation and problem-solving abilities compared to those taught through direct instruction. A statistically significant difference was found in problem-solving ability ($p = 0.001$), while representational ability showed a positive but nonsignificant trend. Moreover, a significant positive correlation ($r = 0.367$, $p = 0.003$*) was found between students' multiple representation and problem-solving abilities. These findings indicate that open-ended learning promotes higher-order thinking by enabling students to explore diverse solution strategies, construct meaning through multiple representations, and engage in reflective reasoning. The study highlights the pedagogical potential of open-ended learning as an effective approach to fostering conceptual understanding and creative mathematical thinking aligned with the goals of the *Merdeka Curriculum*.

Keywords
Open-ended learning; multiple mathematical representation; problem-solving ability; higher-order thinking

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INTRODUCTION

Mathematics plays a crucial role in education, particularly at the junior high school level. Its significance lies not only in supporting other scientific disciplines but also in cultivating logical, analytical, and systematic thinking essential for solving real-life problems (Kilpatrick, Swafford, & Findell, 2001). Students with a strong conceptual understanding of mathematics are more capable of applying their knowledge to academic and practical problem-solving (Hiebert & Grouws, 2007). In accordance with the Merdeka Curriculum, mathematics learning is expected to move beyond procedural routines and promote critical and creative thinking in contextual situations (Artigue, 2009).

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One essential competence to be developed in mathematics learning is mathematical representation ability, namely the ability to express mathematical ideas through various forms—visual, symbolic, and verbal (Ratumanan et al., 2022; Pape & Tchoshanov, 2010). Effective use of representation enables students to concretize abstract concepts and link interrelated ideas (Clements & Sarama, 2010). Research has shown that representation skills strongly correlate with improved mathematical achievement (Maulida et al., 2024) and serve as a foundation for deep mathematical reasoning (Dreyfus, 1999). Another crucial aspect is mathematical problem-solving ability, which reflects students' capacity to devise strategies and solutions for complex and non-routine problems (Wahyuningrum et al., 2024). This ability nurtures higher-order thinking skills—critical, reflective, and creative thinking—that are central to mathematics learning (Fatahillah et al., 2024).

However, classroom practices often fail to align with these educational goals. Preliminary observations in a junior high school in Medan revealed that mathematics instruction remains predominantly teacher-centered. Teachers tend to rely on lectures, examples, and textbook exercises, leaving limited opportunities for students to explore, represent, and communicate their ideas. As a result, students tend to memorize procedures without understanding underlying concepts. This issue aligns with findings by Gebremeskel et al. (2025), who discovered that conventional teaching methods promote passive learning and diminished engagement in mathematical reasoning. In a study conducted in 2018, Yuanita, Zulnaidi, and Zakaria demonstrated that mathematical representations can significantly enhance problem-solving capabilities (Yuanita et al., 2018).

These findings highlight a research gap between the intended goals of mathematics education—which emphasize higher-order thinking (NCTM, 2000; Kilpatrick et al., 2001)—and the limited instructional strategies currently employed to foster representational and problem-solving abilities. While numerous studies have examined the use of problem-based or cooperative learning models, research focusing on the integration of open-ended strategies to simultaneously enhance multiple mathematical representations and problem-solving skills remains scarce, particularly in the context of Indonesian junior high schools.

To address this gap, the present study explores the implementation of the open-ended learning strategy, an approach developed in Japan in the 1970s that encourages students to explore diverse solution methods and representations for a given problem. This strategy aligns with constructivist principles that emphasize active, meaningful learning through exploration, justification, and generalization. Previous studies indicate that open-ended tasks not only enhance students' mathematical achievement but also foster creativity, confidence, and engagement in learning (Desi et al., 2025).

Therefore, this study aims to investigate junior high school students' multiple mathematical representation and problem-solving abilities through the application of the open-ended learning strategy. The findings are expected to contribute to the development of more effective instructional practices that foster students' higher-order mathematical thinking. Furthermore, this study provides practical implications for mathematics teachers in designing learning environments that are more engaging, exploratory, and aligned with the objectives of the Merdeka Curriculum.

METHOD

This study employed a quasi-experimental method with a Pretest–Posttest Control Group Design, which enables objective comparison of students' learning outcomes between those who received the treatment and those who did not. This design was chosen because it allows the researcher to measure changes in students' mathematical abilities before and after the intervention while controlling for initial differences in performance. The population of the study consisted of all eighth-grade students at SMP Negeri 28 Medan during the 2024/2025 academic year. From this population, two intact classes were selected purposively to ensure comparable initial ability levels. Class VIII-1 was assigned as the experimental group, which received instruction using the open-ended learning strategy, while class VIII-2 served as the control group, which received conventional direct instruction.

The research involved one independent variable, namely the application of the open-ended learning strategy, and two dependent variables: students' multiple mathematical representation ability and mathematical problem-solving ability. These variables were measured using three instruments developed and validated specifically for this study. The first instrument was a test of multiple mathematical representation, designed to assess students' ability to express mathematical ideas in visual, symbolic, and verbal forms. The second instrument was a problem-solving test that evaluated students' ability to construct strategies and generate solutions for complex, non-routine mathematical problems. The third instrument was an observation sheet used to record the learning process and student engagement during classroom activities. All instruments were reviewed by experts in mathematics education and piloted to ensure content validity, reliability, and suitability for use in the target population.

The study was conducted in three stages: preparation, implementation, and data analysis. The preparation stage included the design of lesson plans based on the open-ended approach, the development and validation of research instruments, and the pilot testing of these instruments. The implementation stage began with the administration of pretests to both groups, followed by the delivery of instruction according to their respective learning models. The experimental group was taught using open-ended tasks that encouraged multiple representations and diverse solution methods, whereas the control group followed conventional teacher-centered instruction focused on procedural problem solving. At the end of the instructional period, posttests were administered to both groups to assess learning gains. The final stage involved data analysis, which aimed to determine the effectiveness of the open-ended learning strategy and the relationship between students' representational and problem-solving abilities.

Data obtained from the pretests and posttests were analyzed quantitatively using several complementary techniques. Descriptive statistics—including the mean, minimum, maximum, and standard deviation—were used to summarize students' performance. The normalized gain (N-gain) score was then calculated to measure the extent of improvement between the pretest and posttest, categorized into high, medium, and low levels following Hake's (1998) classification. To ensure the validity of inferential testing, normality was assessed using both the Kolmogorov–Smirnov and Shapiro–Wilk tests. Given that the data showed mixed normality results, both parametric and non-parametric analyses were employed. The independent samples t-test was used for normally distributed data, while the Mann–Whitney U test was applied when normality assumptions were not met. These tests compared posttest and N-gain scores between the two groups to determine the significance of differences in learning outcomes. In addition, Spearman's rank correlation analysis was used to examine the relationship between students' multiple mathematical representation ability and their problem-solving ability. The significance level for all tests was set at $p < 0.05$.

To support the interpretation of quantitative findings, observational data were also analyzed descriptively to capture patterns of student engagement and interaction during the learning process. This triangulation between test results and classroom observations provided a more comprehensive understanding of how the open-ended learning strategy contributed to the development of students' representational and problem-solving competencies. Overall, this methodological framework was designed to rigorously assess both the effectiveness and the cognitive impact of the open-ended approach in mathematics learning.

RESULTS AND DISCUSSION

The findings of this study describe the effects of the open-ended learning strategy on students' multiple mathematical representation and problem-solving abilities, as well as the relationship between the two variables. The results are presented based on descriptive statistics, inferential tests, and correlation analysis. The descriptive results of students' multiple mathematical representation ability are presented in Table 1.

Table 1. Descriptive statistics of multiple mathematical representation ability

Learning strategy	Test type	Max	Min	$\bar{x}$	Stdev
Open-ended	Pre-test	26	9	15.97	4.76
	Post-test	66	23	38.55	12.11
Direct instruction	Pre-test	26	10	15.55	4.33
	Post-test	47	22	34.4	8.55

The data indicate that both groups experienced an increase in scores from pretest to posttest, but the improvement was more substantial for the open-ended learning group (Δ = 22.58) compared to the direct instruction group (Δ = 18.85). This result demonstrates that the open-ended learning strategy provided greater opportunities for students to develop diverse representations of mathematical ideas. The normalized gain (N-gain) analysis in Figure 1 shows that most students in both groups achieved moderate levels of improvement, but only the experimental group reached the high-gain category. This suggests that open-ended learning fostered students' engagement, exploration, and flexibility in representing mathematical concepts, while direct instruction tended to limit student initiative to procedural responses.

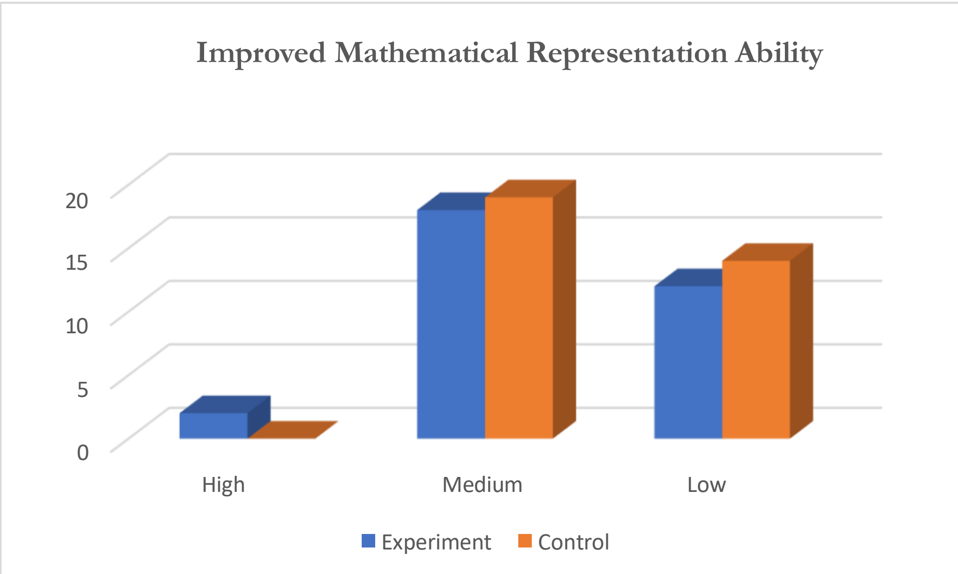

Figure 1. N-Gain of multiple mathematical representation ability

The results of the normality test are presented in Table 2.

Table 2. Tests of normality for multiple mathematical representation ability

Variable	Group	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Pre-test Representation	Experimental	0.164	32	0.028	0.902	32	0.007
Pre-test Representation	Control	0.156	33	0.040	0.954	33	0.172
Post-test Representation	Experimental	0.182	32	0.009	0.922	32	0.024
Post-test Representation	Control	0.174	33	0.013	0.900	33	0.005

Most data were not normally distributed ($p < 0.05$), except for the pretest in the control group ($p = 0.172$). Therefore, the Mann–Whitney U test (see Table 3) was used to compare posttest results between groups.

Table 3. Mann–Whitney U test for posttest of multiple representation ability

Test statistics	
	Post-tets Representation
Mann–Whitney U	415.500
Wilcoxon W	976.500
Z	-1.485
Asymp. Sig. (2-tailed)	0.137

The test result ($p = 0.137 > 0.05$) indicated no statistically significant difference between groups. However, the experimental group had a higher mean rank (36.52) than the control group (29.59), implying a positive tendency that the open-ended strategy enhanced students' ability to use multiple mathematical representations more effectively, even though the difference was not statistically significant. Regarding mathematical problem-solving ability, descriptive statistics are shown in Table 4.

Table 4. Descriptive statistics of mathematical problem-solving ability

Learning strategy	Test type	Max	Min	$\bar{x}$	Stdev
Open-ended	Pre-test	16	4	9.38	2.80
	Post-test	39	16	27.88	7.10
Direct instruction	Pre-test	16	5	9.91	2.75
	Post-test	36	11	21.91	6.18

Students in the open-ended learning group achieved a higher average posttest score (M = 27.88) than those in the control group (M = 21.91). This result demonstrates that the open-ended learning strategy was more effective in enhancing students' ability to construct and implement strategies for solving mathematical problems. Figure 2 illustrates that students in the experimental group reached higher improvement categories on average, with several achieving high gains.

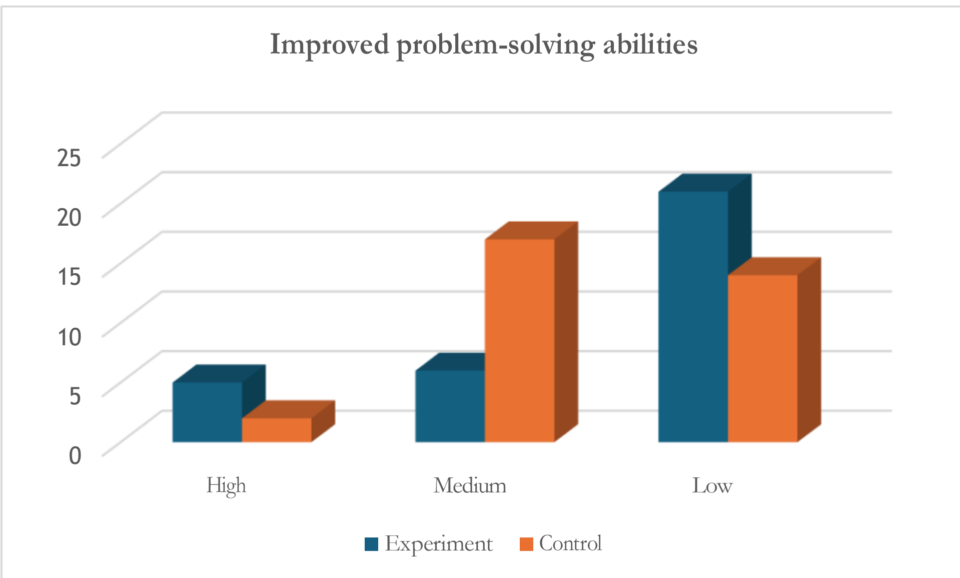

Figure 2. N-Gain of mathematical problem-solving ability

Conversely, most students in the control group were in the medium category, and some experimental group students were in the low category, possibly due to unfamiliarity with the open-ended approach. This finding reinforces that open-ended learning supports cognitive and reflective development through exploration and diverse problem-solving strategies. The results of the normality test for problem-solving ability are presented in Table 5.

Table 5. Tests of normality for mathematical problem-solving ability

Variable	Group	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Pre-test Problem-solving	Experimental	0.156	32	0.046	0.961	32	0.290
Pre-test Problem-solving	Control	0.134	33	0.141	0.968	33	0.432
Post-test Problem-solving	Experimental	0.126	32	0.200	0.939	32	0.071
Post-test Problem-solving	Control	0.095	33	0.200	0.962	33	0.296

The Shapiro–Wilk test results show that the data were normally distributed ($p > 0.05$). Therefore, the independent samples t-test was used to analyze posttest differences between groups. Subsequently, the results of the statistical test assessing students' mathematical problem-solving capabilities are presented in Table 6.

Table 6. Independent samples t-test for mathematical problem-solving ability

		Levene's test for equality of variances						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. error difference
Problem-solving	Equal variances assumed	0.909	0.344	3.616	63	0.001	5.96591	1.64992
	Equal variances not assumed			3.608	61.253	0.001	5.96591	1.65348

The result of the independent samples t-test indicated a significant difference between the two groups ($p = 0.001 < 0.05$). This means that students who were taught using the open-ended strategy demonstrated significantly higher problem-solving ability than those taught using direct instruction. The relationship between multiple mathematical representation and problem-solving abilities was further examined using Spearman's rank correlation, as presented in Table 7.

Table 7. Spearman's rank correlation between multiple representation and problem-solving abilities

Variable		Post-test Representation	Post-test Problem-solving
Post-test Representation	Correlation coefficient	1.000	0.367**
	Sig. (2-tailed)		0.003
	N	65	65
Post-test Problem-solving	Correlation coefficient	0.367**	1.000
	Sig. (2-tailed)	0.003	
	N	65	65

Note. Correlation is significant at the 0.01 level (2-tailed).

The analysis revealed a significant positive correlation between multiple representation ability and problem-solving ability ($r = 0.367$, $p = 0.003$). This finding suggests that students with higher flexibility in representing mathematical ideas also tend to perform better in solving complex problems.

Overall, the results indicate that the open-ended learning strategy was more effective than direct instruction in enhancing students' mathematical abilities. Although the difference in representational ability was not statistically significant, the experimental group consistently demonstrated higher mean scores and stronger improvement trends. For problem-solving ability, the difference was statistically significant, highlighting that open-ended learning facilitates critical, creative, and reflective thinking. Moreover, the moderate positive correlation between both variables confirms that the development of representational skills supports and reinforces students' capacity for mathematical problem solving.

The findings of this study reveal that the open-ended learning strategy effectively enhances students' mathematical problem-solving ability and shows a positive trend in improving multiple mathematical representation ability. Although the difference in representational ability between groups was not statistically significant, the experimental group consistently outperformed the control group. In contrast, the difference in problem-solving ability was statistically significant, indicating that the open-ended approach fosters higher-order thinking processes such as reasoning, creativity, and reflective judgment.

The observed improvement in students' multiple mathematical representation ability can be attributed to the open-ended nature of the learning environment, which allows students to express mathematical ideas in various forms—symbolic, visual, and verbal. This flexibility facilitates deeper conceptual understanding and helps bridge abstract and concrete mathematical ideas. As Dreyfus (1999) explains, representation acts as a mediator in mathematical reasoning, enabling learners to construct meaning and connect different aspects of mathematical knowledge. Likewise, Fonger (2019) emphasize that representational fluency supports students' capacity to translate, interpret, and interrelate mathematical ideas across contexts. Recent studies such as Awaludin et al. (2021) and Fatqurhohman and Susetyo (2022) further confirm that students who employ varied representational strategies—such as symbolic, pictorial, and verbal—tend to demonstrate higher levels of comprehension and problem-solving accuracy.

The open-ended approach used in this study allowed students to explore multiple solution pathways and discuss diverse representations collaboratively. This finding resonates with Nohda's (2000) assertion that open-ended problem situations encourage students to develop creative mathematical thinking by constructing and sharing their own understanding. Moreover, Kurniati and Sutiarso (2021) found that open-ended learning environments enhance higher-order reasoning and foster persistence in tackling non-routine problems. In a similar vein, Fatah et al. (2016) reported that the open-ended approach nurtures mathematical creativity, communication, and self-confidence—key dispositions required in modern mathematics education. These findings align with Artigue (2009), who underscores that meaningful mathematics learning involves exploration, argumentation, and justification within a supportive classroom culture.

The significant difference in problem-solving ability between the experimental and control groups indicates that the open-ended learning strategy provides a more conducive environment for developing students' strategic competence. Students in the experimental group were encouraged to devise, compare, and evaluate multiple strategies to reach a solution, which aligns with Kilpatrick, Swafford, and Findell's (2001) framework of mathematical proficiency. According to Hiebert and Grouws (2007), problem solving is most effective when students engage in reasoning that connects procedures with underlying concepts, a condition that the open-ended strategy naturally promotes. Empirical findings from Johar and Lubis (2018) and Ruamba et al. (2023) also support this notion, showing that open-ended and problem-based strategies lead to substantial gains in both representational and problem-solving competencies among middle school students.

The correlation analysis revealed a moderate but significant relationship between multiple representation and problem-solving abilities ($r = 0.367$, $p = 0.003$*), suggesting that representational competence contributes meaningfully to students' success in problem solving.

This is consistent with Ainsworth's (2006) DeFT framework, which posits those multiple representations support learning by complementing, constraining, and constructing understanding. Students who can flexibly shift among visual, algebraic, and verbal forms of representation can better analyze problems, verify results, and communicate reasoning effectively (Liani, 2023; Nurrahmawati et al., 2019; Muniri & Musrikah, 2024). Similarly, Nasrun et al. (2023) demonstrated that representational fluency predicts the ability to generalize mathematical structures and transfer knowledge to unfamiliar problem contexts.

In the current study, the open-ended learning environment also promoted active participation and peer interaction. Students were encouraged to justify their reasoning and evaluate others' strategies, which contributed to reflective and metacognitive thinking. Such interactional dynamics align with Maries, Lin, and Singh (2020), who observed that students' representational consistency depends on their epistemological framing during collaborative problem-solving tasks. In this regard, open-ended learning not only facilitates individual cognition but also cultivates collective reasoning and communication skills essential for mathematical literacy (Dewi & Juandi, 2024).

The absence of a statistically significant difference in multiple representation ability between groups, despite the higher mean score in the experimental class, may stem from students' initial unfamiliarity with open-ended problem contexts. As Al Mamun and Lawrie (2024) explain, learners transitioning from conventional to exploratory environments often require scaffolding to adapt to new expectations of autonomy and inquiry. This interpretation is also supported by Shofia et al. (2020), who found that students' representational development in mathematics depends heavily on sustained exposure to tasks that demand conceptual reasoning rather than procedural recall. Thus, to maximize the benefits of open-ended learning, consistent implementation and teacher facilitation are critical.

Overall, this study reinforces the view that the open-ended learning strategy is an effective pedagogical approach to promoting higher-order mathematical thinking. The strategy provides opportunities for students to develop representational fluency, strategic competence, and self-regulated learning—key components of mathematical proficiency in the 21st century. These findings are in line with the Merdeka Curriculum's emphasis on inquiry-based and student-centered learning. Consequently, integrating open-ended problem situations in mathematics classrooms can bridge the gap between procedural mastery and conceptual understanding, fostering learners who are capable, creative, and confident in reasoning mathematically.

CONCLUSIONS

The findings of this study indicate that the implementation of the open-ended learning strategy in mathematics instruction positively influences students' learning outcomes, particularly in enhancing their problem-solving ability and promoting multiple mathematical representation. Although the improvement in representational ability was not statistically significant, students exposed to open-ended learning consistently demonstrated higher mean scores and richer representational forms than those in direct instruction settings. The difference in problem-solving ability was statistically significant, showing that the open-ended learning strategy effectively stimulates higher-order cognitive processes such as reasoning, creativity, and reflective thinking. Furthermore, the significant positive correlation between multiple representation and problem-solving ability suggests that students' capacity to express mathematical ideas through various representations strengthens their problem-solving performance.

These results confirm that open-ended learning provides meaningful opportunities for students to engage in exploratory reasoning, articulate diverse strategies, and develop conceptual understanding through active participation. In this sense, open-ended learning not only supports individual cognitive growth but also promotes collaboration, metacognitive awareness, and communication of mathematical ideas. This finding aligns with the principles of the Merdeka Curriculum, which emphasizes inquiry-based, student-centered, and competency-oriented learning.

From a practical perspective, the results of this study imply that teachers should integrate open-ended problem situations into mathematics lessons more systematically. Through this approach, teachers can create learning environments that encourage students to explore multiple strategies, justify their reasoning, and reflect on the efficiency and elegance of their solutions. To optimize its effectiveness, teachers also need to provide scaffolding that helps students transition from procedural dependence toward autonomous exploration, especially for learners who are less familiar with open-ended tasks.

From a theoretical standpoint, this research reinforces the notion that multiple mathematical representations are foundational to problem-solving proficiency. It supports the idea that representation and problem solving are interdependent processes that mutually reinforce one another. Accordingly, future studies should further investigate the longitudinal effects of open-ended learning on metacognitive regulation, creativity, and reasoning structures across different mathematical domains. Additionally, employing mixed-method or design-based research approaches could provide deeper insights into how open-ended learning shapes students' cognitive and affective engagement over time.

In conclusion, the open-ended learning strategy holds strong potential as a transformative pedagogical model for mathematics education. It bridges the gap between procedural mastery and conceptual understanding, cultivates representational fluency and problem-solving competence, and fosters students who are capable, creative, and reflective mathematical thinkers.

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