2025, 6(3), 207–216

Journal of Didactic Mathematics
https://doi.org/10.34007/jdm.v6i3.2903

Exploring reversible thinking through comparison task in mathematical praxeology textbooks

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Abstract.
The capacity for reversible thinking is a fundamental aspect of proficient mathematical problem-solving. However, existing research indicates that students continue to encounter challenges in cultivating this cognitive process. One contributing factor to this difficulty is the inclination of textbook tasks to prioritize procedural learning over conceptual exploration. The objective of this study was to examine the task sequence structure in seventh-grade mathematics textbooks on the subject of comparison, specifically in two primary tasks: comparing two similar quantities and comparing two quantities with differing units. The textbook analysis technique employs a mathematical praxeology approach. The analysis encompasses four components of praxeology: tasks, techniques, technology, and theory. The textbook utilized is Mathematics, Grade 7, junior high school, Semester 2. The findings reveal that the majority of problem-solving techniques are presented directly within the textbook, thereby restricting students' opportunities to develop their own strategies, particularly reversible thinking strategies. Furthermore, the majority of tasks are designed to promote forward thinking, thereby limiting students' opportunities to develop two-way thinking skills. To address this issue, the study recommends formulating an alternative sequence of tasks that explicitly encourages the development of reversible thinking strategies.

Keywords:
Reversible thinking; mathematics textbooks; mathematical praxeology; series of task; learning obstacle

How to cite:
Pebrianti, A., Darojat, L., & Heryanto, F. N. (2025). Exploring reversible thinking through series of task in mathematical praxeology textbooks. *Journal of Didactic Mathematics*, *6*(3), 207–216. https://doi.org/10.34007/jdm.v6i3.2903

INTRODUCTION

Reversible thinking is a crucial component of students' development of mathematical thinking skills, particularly in the context of problem-solving. Reversible thinking can be defined as the ability to reconstruct a process or steps of solution from the result back to the initial data or initial conditions of a problem (Ramful, 2015; Maf'ulah & Juniati, 2020). This ability is indicative of a profound conceptual understanding, as students demonstrate an ability to not only comprehend the procedures but also to discern and articulate the relationships between the mathematical elements involved in a given problem.

In the realm of mathematics education, reversible thinking has demonstrated its efficacy in enhancing problem-solving proficiency and serving as a pivotal indicator of students' conceptual comprehension (Pebrianti et al., 2022; Pebrianti et al., 2023). This cognitive process necessitates that students cultivate the capacity to perceive a problem from two distinct viewpoints: from data to solution (forward thinking) and from solution to data (backward thinking). This aptitude holds significant relevance across a diverse array of mathematical domains, encompassing ratios, algebra,

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Received: 2 October 2025; Revised: 25 November 2025; Accepted: 31 December 2025
2721-5601/ © 2025 The Author(s). Published by Mahesa Research Center

fractions, and calculus, as it necessitates a logical reciprocal relationship between the mathematical elements employed (Saparwadi et al., 2020; Steffe & Olive, 2009).

Despite the numerous studies conducted, the findings suggest that the reversible thinking skills of Indonesian students remain relatively underdeveloped. For instance, Maf'ulah et al. (2019) identified challenges faced by students in establishing reciprocal relationships between function representations and their graphs. Research indicates that this ability does not develop optimally across various educational levels, from elementary school to university (Ramful, 2015; Robinson & LeFevre, 2012). The limited efficacy of this ability can be attributed to a predominantly procedural learning approach that provides minimal opportunities for exploring alternative thinking strategies, such as backward thinking.

The current study posits that one of the contributing factors to diminished reversible thinking ability is the learning impediments encountered by students during the mathematics learning process. (Brousseau, 2006) delineates these learning obstacles into three distinct categories: (1) epistemological obstacles, which pertain to limitations inherent in mathematical knowledge; (2) ontogenically obstacles, which relate to students' cognitive and psychological development; and (3) didactic obstacles, which stem from the manner in which material is presented by instructors or textbooks. Within this framework, textbooks assume the role of primary learning resources within the classroom, functioning as intermediaries between students and mathematical concepts.

Mathematics textbooks often present routine and procedural tasks, neglecting to provide opportunities for students to develop more flexible and profound thinking strategies. Textbooks that do not offer a variety of rich tasks and do not support two-way exploration in thinking can hinder the development of students' reversible thinking (Jäder et al., 2020; O'Sullivan et al., 2024; Yunianta et al., 2023). Consequently, a critical review is imperative to assess the organization of tasks in mathematics textbooks, with a particular focus on whether these tasks facilitate two-way logical thinking (Fan et al., 2025; Kaur & Chin, 2022; Star et al., 2022).

One approach to analyzing the structure of tasks in textbooks is mathematical praxeology, a concept in didactic anthropology developed by (Chevallard, 1992). Mathematical praxeology comprises four primary components: task (T), which denotes the activity that must be performed; technique (τ), which refers to the method or procedure for completing the task; technology (θ), which signifies the rationale or justification for employing that technique; and theory (Θ), which pertains to the broader mathematical principle or law underlying that technology. The integration of these four components yields a comprehensive analysis of the manner in which mathematical activities are constructed within a learning context.

The praxeological approach empowers researchers and educators to delve into the intricacies of student actions, encompassing not only their behaviors but also the underlying concepts and motivations that drive them. Within the realm of reversible thinking, praxeological analysis can elucidate whether a task facilitates students' utilization of specific techniques in a reciprocal manner, and whether the task incorporates explanations of the technology and theoretical underpinnings that support it. Tasks that remain confined to the level of technique, neglecting the development of technology and theory, frequently fall short of fostering profound thinking skills, including reversible thinking.

Furthermore, praxeological studies can assist in identifying discrepancies between the techniques taught and the theoretical justifications necessary for flexible thinking. For instance, if a problem solely necessitates calculation without providing an opportunity to inquire "why" a specific technique is employed, students may not develop a habitual reliance on the conceptual justifications that are critical in reversible thinking. In this case, praxeology functions not only as an analytical tool but also as a guide in redesigning mathematical tasks to make them more meaningful.

The objective of this study is to examine the structure of tasks within seventh-grade mathematics textbooks, specifically the Gatotkaca textbook, to ascertain the extent to which these tasks facilitate or impede students' reversible thinking employing a mathematical praxeological approach. This review is novel due to the absence of praxeological textbook analyses that specifically focus on reversible thinking.

Based on the findings of this analysis, a series of revised tasks will be formulated. This new series will be more congruent with the principles of reversible thinking and supported by robust praxeological justifications.

METHOD

In this study, a praxeological analysis approach is employed to investigate the existence and characteristics of reversible thinking, as evidenced in comparison tasks found within mathematics textbooks. The development of the praxeological framework is closely associated with Chevallard's Didactic Anthropology theory, which conceptualizes mathematical activities as praxeological units comprising four components: type of tasks (T), techniques (τ), technologies (θ), and theories (Θ). This approach facilitates the identification of the structure of mathematical activities that implicitly or explicitly provide opportunities for reversible thinking students.

Data Resource

The primary data source for this study was the Mathematics Grade VII SMP/MTs Semester 2 textbook published by the Ministry of Education and Culture of the Republic of Indonesia (2017, revised edition). The selection of this book was motivated by its status as a reference text utilized by one of the schools in Bandung Regency. Notably, it has achieved national recognition, becoming a widely adopted resource in Indonesian junior high schools. This underscores its significance in the broader context of the national curriculum, particularly with respect to promoting reversible thinking.

Data Collection Technique

Data collection was conducted by extracting all comparison tasks. Comparison tasks are defined as questions that necessitate students to compare mathematical objects, representations, procedures, or results on the topics of Understanding the Comparison of Two Quantities and Determining the Ratio of Two Quantities with Different Units. Each task that meets these criteria was systematically documented, classified, and copied in the analysis sheet.

Data Analysis

Data analysis was conducted by identifying the praxeological structure of each comparison task in the textbook. This included the type of task (T), expected technique (τ), underlying technology (θ), and related mathematical theory (Θ). Each task was then coded based on reversible thinking indicators adapted from the literature. These indicators included the need to reverse operations, perform backward thinking, and comprehend the bidirectional relationship between representations. Subsequently, mapping was performed between the praxeological structure and the reversibility category to assess the extent to which the techniques and technologies provided by the textbook facilitated the development of reversible thinking. Additionally, patterns of support or didactic limitations that emerged in the task design were identified.

RESULTS AND DISCUSSION

The identification of learning obstacles in students' problem-solving abilities on the topic of comparison, which necessitates reversible thinking skills, was conducted through an in-depth study of a series of tasks in mathematics textbooks using Chevarald's praxeology technique.

The sequence of tasks in textbooks holds significant importance as it serves as an additive component that aids students in constructing their mindset and developing their knowledge base. Prior to conducting the study, a selection of the textbook was made based on the results of interviews with mathematics teachers. The school in question utilizes a mathematics textbook titled "Mathematics Grade VII SMP/MTs Semester 2 Ministry of Education and Culture of the Republic of Indonesia 2017 (revised edition)." Relevant topics for research can be found in chapter

5 of the book in question. Permission to study the textbook was granted based on a statement on the "Foreword" page, which states that: This book remains a work in progress, necessitating ongoing improvement and refinement. Consequently, we extend an invitation to the readership to submit critiques, recommendations, and observations that will contribute to the enhancement and refinement of subsequent editions. We extend our gratitude for your contributions. It is our objective to contribute to the advancement of education in preparation for the centennial of Indonesia's independence (2045) to the best of our abilities. This statement enables researchers to conduct studies on this textbook, with the objective of refining the content presented, particularly in the comparison materials section.

In the comparison chapter of the 2017 revised edition of the mathematics textbook, there are five types of tasks, which include: (1) understanding and determining the comparison of two quantities; (2) determining the comparison of two quantities with different units; (3) understanding and solving problems related to equivalent comparisons; (4) solving problems involving direct ratios on maps and models; and (5) understanding and solving problems related to inverse ratios. Of the five types of tasks presented in the book, only two types of tasks were analyzed, namely the first and second types of tasks. This conclusion was derived from the results of interviews in which respondents indicated that these two types of tasks played an important role in building concepts in comparative material. The researcher's analysis of the textbook centered on the series of tasks that shaped students' knowledge of comparative concepts. The present research analysis does not concentrate on tasks outside the sequence, such as practice problems. As illustrated in Table 1, the results of the evaluation of the sequence of tasks in the mathematics textbook on the topic of comparison are shown. This evaluation was based on the four elements of praxeology. The italicized sentences are guiding sentences found in the book, while the sentences written in normal font are the author's interpretations. Subsequently, the $t_{1.1}$ code signifies the initial task in the series of task designated as one. Consequently, $t_{2..1}$ designates the initial task in series of task two, while $t_{2.2}$ signifies the subsequent task in the same series of task. This rule is applicable to τ, θ, and Θ as well, thus yielding equivalent interpretations.

Table 1. A praxeological analysis of task sequences in textbooks

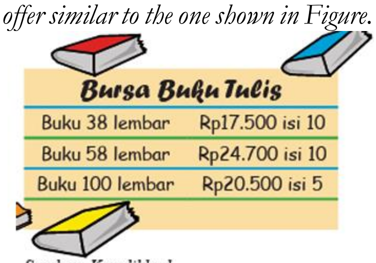
Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
Task 1: Understanding the Comparison of Two Quantitie			
$t_{1.1}$ At the beginning of the comparison lesson, students will participate in Activity 5.1. Activity 5.1 involves an image representing an event. The following is presented in the book. *The story is about a family of seven women and nine men who are taking a photo on the beach.*  *Sumber: Kemdikbud*	$\tau_{1.1}$ The book explains the ratio between the number of women and men, so students cannot use their own techniques based on the gender of their family members to explain the ratio. The following techniques are presented in the book. *Nadia told her friends about the photo as follows:* *1. Seven of the sixteen people in the photo are women.* *2. The ratio of men to women is nine to seven.* *3. There are two more men than women.*	$\theta_{1.1}$ Although the book presents a technique for solving problem 5.1, it still allows students to explain or justify the answers provided. However, the book does not guide students step by step, so they may find it difficult to develop the expected concepts. *In your opinion, what is the best way to express the ratio of men to women in Nadia's family photo? Why?*	$\Theta_{1.1}$ We will introduce th e concept of comparison by comparing two similar components.
Task 2: Determining the Ratio of Two Quantities with Different Units			

<table>
<tr><th>Task (T)</th><th>Technique (τ)</th><th>Technology (θ)</th><th>Theory (Θ)</th></tr>
<tr>
<td>$t_{2.1}$
To begin learning about the second topic, students observe events involving comparisons. The following are some of the events described in the book.

The following example illustrates another way of comparing numbers:
- A nutrition label states that four biscuits contain 100 kcal of energy.


Sumber: Kemdikbud
My father's motorcycle can travel 40 kilometers on one liter of Pertamax fuel on smooth roads.</td>
<td>$\tau_{2.1}$
The book presents the technique for completing task T2.2 in full, leaving no room for students to develop their own techniques. The following are the techniques presented in the book.

Which of the six statements above is different? Each statement compares two different quantities. For example, it compares distance traveled (kilometers) with amount of Pertamax (liters), internet rate per hour, rupiah exchange rate against the dollar, and speed.</td>
<td>$\theta_{2.1}$
Students are not given the opportunity to provide justification or reasons. However, based on the review of the material and the series of tasks in Task 1, students should understand the ($\tau_{1,1}$) technique described in the book. This is because they already have experience learning about comparative forms.</td>
<td>$\Theta_{2.1}$
Comparing two equivalent statements with different units.</td>
</tr>
<tr>
<td>$t_{2.2}$
The next topic for comparison in $t_{2.2}$ is:

One day, Hardianto came across an offer similar to the one shown in Figure.


Sumber: Kemdikbud
The prices listed were for five, ten, and twelve books. Another way to present the prices is in the form of a table, such as the one below</td>
<td>$\tau_{2.2}$
Students do not have the opportunity to express their own techniques for completing task $t_{2.2}$ because the book provides all of the techniques. The provided techniques calculate the prices of the books from smallest to largest. The following techniques are presented in the book (in rupiah).
<table>
<tr><th>Banyak buku</th><th>1</th><th>2</th><th>5</th><th>10</th><th>12</th></tr>
<tr><td>Buku 38 lb (A)</td><td>1.750</td><td>3.500</td><td>8.750</td><td>17.500</td><td>21.000</td></tr>
<tr><td>Buku 50 lb (B)</td><td>2.470</td><td>4.940</td><td>12.350</td><td>24.700</td><td>29.640</td></tr>
<tr><td>Buku 100 lb (C)</td><td>4.100</td><td>8.200</td><td>20.500</td><td>41.000</td><td>49.200</td></tr>
</table></td>
<td>$\theta_{2.2}$
Students are not given the opportunity to explain why they would use the techniques presented in the book. Additionally, the book does not explain why the given answers are correct.</td>
<td>$\Theta_{2.2}$
Comparing Two Equivalent Statements with Different Units</td>
</tr>
<tr>
<td>$t_{2.3}$
The next topic for comparison in $t_{2.3}$ is:

Agung cycled on different tracks. Sometimes he rode uphill, and sometimes he rode downhill. Sometimes he rode on flat roads. He stopped three times to record the time and distance traveled after passing three tracks.
• Stop 1: 8 kilometers; 20 minutes
• Stop 2: 12 kilometers; 24 minutes</td>
<td>$\tau_{2.3}$
Students do not have the opportunity to express their own techniques for completing task $t_{2.3}$ because the book provides the technique entirely. The provided technique calculates the price of the book from smallest to largest. The technique presented in the book is shown below (in rupiah).</td>
<td>$\theta_{2.3}$
Students are not given the opportunity to explain why they would use the techniques presented in the book. Additionally, the book does not explain why the given answers are correct</td>
<td>$\Theta_{2.3}$
Comparing two equivalent statements that have different units.</td>
</tr>
</table>

Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
On which route did Agung ride his bike quickly?	*First, we must determine Agung's average speed on each track. On the first track, Agung traveled eight kilometers in 20 minutes. Therefore, he rode his bicycle at a speed of* $\frac{8}{20}=\frac{2}{5}=\frac{4}{10}$ *km/minute. On the second track, Agung traveled 12 kilometers in 24 minutes. Therefore, he rode his bicycle at a speed of* $\frac{12}{24}=\frac{1}{2}=\frac{5}{10}$ *km/minute. Since* $\frac{2}{5}<\frac{1}{2}$, *Agung rode his bike fastest on the second track.*		

It is important to note that each task type (T) comprises multiple tasks that build upon the primary task. Each task (t) is meticulously designed to facilitate students in achieving their stipulated learning objectives. In task type 1 (T_1), the objective is to facilitate students' comprehension of the comparison concept. The tasks are structured into a sequence, designated as $t_{1.1}$. The task of Task $t_{1.1}$ is designed to assist students in the construction of mathematical sentences, integrating both sentences and images. In such instances, the image employed to elucidate the intended meaning of the sentence can be refined by eliminating the presence of individuals in the background of the photograph. This phenomenon can be attributed to the potential disruption it might cause in the process of enumerating the total number of individuals depicted in the image, as it could lead to a slight confusion among the students.

In Task Type 2 (T_2), students are introduced to the concept of comparing two equivalent quantities with differing units. This task involves comparing two components that possess equivalent values but are measured in different units.

For Task $T_{2.1}$, several everyday examples are presented with three problems. These problems illustrate that comparisons can be made using different units. However, the image in the first problem related to nutritional information (Table 1) is unclear and may pose a challenge for students to comprehend. It would be more effective to replace the image with a visual representation of the sugar required to prepare one batch of cake batter, for instance.

In Task $T_{2.2}$, students are guided in identifying the concept of comparison between two components with different units but equal values. The problem presented involves the pricing of books in various quantities, ranging from one book to twelve books. It would be more convenient for students to perform division and multiplication if the book prices were expressed in base-10 numbers. Additionally, it is advisable to adjust the book prices to reflect actual market prices.

For Task $T_{2.3}$, students are expected to independently establish equivalent comparisons. It is assumed that students have completed Task $T_{2.2}$ before attempting this task. However, if they have not fully grasped the concepts covered in Task $T_{2.2}$, they may encounter difficulties in completing Task $T_{2.3}$. Therefore, providing guidance to help them understand equivalent comparisons is crucial.

In contrast, the book explicitly presents nearly all techniques, leaving no room for students to contribute techniques based on their own perspectives. Furthermore, the tasks presented do not guide students in constructing reversible thinking strategies. However, certain types of tasks can guide students in developing reversible thinking patterns. One example is task types $t_{2.3}$, which are designed to develop the ability to compare two quantities with different units. The book explains examples of comparisons by directly showing the form of the comparison or directly showing the correct answer to students. However, it would be more beneficial if students discovered the form of the comparison on their own.

Subsequently, task type 2 ($t_{2.2}$) presents a presentation comprising problems pertaining to book pricing, which are organized in rows and columns. If the task provides ample space, students are instructed to determine the quantity of books purchased at the specified price. Subsequently, they are guided through a reversible thinking process, wherein they are provided with the final result (book price) and tasked with identifying the initial value (number of books). Figure 1 illustrates the original tasks encountered within the book, followed by recommendations on how these tasks can effectively motivate students to actively engage in problem-solving and foster the development of reversible thinking.

Contents of mathematics textbooks

Banyak buku	1	2	5	10	12
Buku 38 lb (A)	1.750	3.500	8.750	17.500	21.000
Buku 50 lb (B)	2.470	4.940	12.350	24.700	29.640
Buku 100 lb (C)	4.100	8.200	20.500	41.000	49.200

Suggestions for dishes that require reversible thinking skills

Banyak buku	1	3	b	c	10
Buku 38 lembar	2.500	7.500	12.500	20.000	25.000
Buku 50 lembar	3.300	a	16.500	d	33.000

Figure 1. Suggestions for tasks that promote active problem-solving and foster flexible thinking in pupils

In addition to enabling students to explore techniques, the provided instructions must support their implementation. For instance, in the provided example, students are initially prompted to determine the value of a employing a forward-thinking strategy: "If one book is purchased for 3,300, how many books can be acquired for 3,300?" Subsequently, upon successfully completing the initial problem, students are directed to ascertain the value of b utilizing a reversible thinking strategy: "If one book incurs a cost of 1,500, how many can be purchased for 12,500?" This connection extends beyond task $t_{2.2}$, as the reversible thinking process can be seamlessly integrated into other tasks. The justification or explanation of the technique employed is commonly referred to as technology in praxeology. Certain mathematics textbooks necessitate an explanation of the technology underlying the technique elucidated within the book. Occasionally, the instructions may be ambiguous, hindering students' ability to substantiate the technique utilized. Following technology is the final component of praxeology: theory. Theory represents the fundamental concept or knowledge that emerges. The comparison presented in the book aligns with the concept of comparison articulated by experts, thereby eliminating any concerns regarding the comprehension of comparison presented in the book. However, upon analyzing the tasks, techniques, and technology, it becomes evident that the sequence of tasks does not effectively construct a reversible thinking process for students. This observation is evident from the outset to the conclusion of the material, as there are no tasks that necessitate students to engage in a reverse thinking process. All tasks are meticulously designed with a forward-thinking approach. In fact, this capacity for reversible thinking is crucial for enhancing students' comprehension of comparisons. Reversible thinking entails the ability to work on tasks in reverse order. Consequently, students will attain a more comprehensive understanding of the material when they can engage in tasks from two perspectives.

A comparative analysis of the problem-solving tasks presented in the Mathematics Grade 7 SMP/MTs Semester 2 textbook reveals that the task structure does not effectively promote student engagement in the problem-solving process (Prabawanto et al., 2023). The majority of tasks were of a procedural nature, providing explicit answers or steps for completion, thereby reducing opportunities for students to explore concepts independently (Asmida et al., 2018; Barumbun & Kharisma, 2022). This pattern of presentation suggests that the praxeological structure proposed by the textbook remains limited to the provision of techniques, without providing space for deeper reflection or mathematical justification. Indeed, certain tasks possess the potential to be transformed into activities that promote reversible thinking. For instance, by instructing students to trace the two-way relationship in ratios, examine the equivalence of representations, or reverse the calculation process, educators can effectively facilitate reversible thinking in their students. The enhancement of the reversibility element in task design has been demonstrated to have a dual benefit: it can improve students' conceptual understanding and enrich the learning experience by engaging them as active participants in constructing mathematical meaning, rather than merely following procedures.

A more comprehensive discussion of each type of task is provided for task Type T_1, which focuses on knowledge related to comparing two quantities in a specific situation. The problems presented involve activities commonly encountered in daily life, enabling students to comprehend the concepts in a practical context. This approach is advantageous for solving mathematical problems as it cultivates sensitivity to mathematical issues that arise in everyday life (Widjaja, 2013). However, the utilization of context through images or illustrations requires precision. Visuals that are unclear or overly complex have the potential to introduce ambiguity in interpretation and hinder the comprehension of the fundamental concepts being developed. Given that the primary objective of T_1 is to establish the foundational principle of comparison, the task structure within this category remains limited to forward-thinking activities. It does not yet encompass reversible thinking, such as tracing the two-way relationship between quantities or reversing the comparison process.

In the subsequent task, designated as Type 2, the primary learning objective shifts from the comprehension of comparisons' meanings to the ability to compare two quantities that are equivalent but presented in different units. Tasks within this category should provide students with opportunities to convert units, explore two-way relationships between quantities, and examine the equivalence of values in various representations. However, the presentation pattern of these tasks mirrors that of Type 1 tasks, wherein textbooks tend to provide explicit final answers or present overly directed solution steps. This approach, however, imposes limitations on students' capacity to explore independently, both in determining conversion strategies and in conducting more in-depth comparative reasoning. Consequently, students encounter significant constraints in their ability to cultivate open-mindedness, re-evaluate the relationship between two quantities, or perform reversible processes such as unit conversion. In essence, while Type 2 exhibits considerable capacity to promote reversible thinking through unit transformation activities and equivalence analysis, the overly constrained task design in textbooks impedes the development of this cognitive process (Jonsson et al., 2020).

This study revealed that nearly all of the presented tasks employed techniques explicitly provided by the book. These processes or workflows did not facilitate the development of knowledge; instead, they offered direct solutions, raising concerns that students would memorize procedures rather than concepts. Several studies have demonstrated that, when learning mathematics, students frequently memorize problem-solving steps rather than comprehending the underlying concepts (Pirmanto et al., 2020). Conversely, students must possess a comprehensive understanding of the concepts to successfully complete tasks that necessitate reversible thinking (Maf'ulah et al., 2019). A solid grasp of concepts supports success in reversible thinking, and vice versa.

Upon analyzing the task components, techniques, and technology presented in the book, it becomes evident that the sequence of tasks does not effectively foster the development of students' reversible thinking abilities. Despite the apparent consistency of the praxeological structure in its organization of step-by-step procedures, the absence of tasks that necessitate students to retrace two-way relationships, reverse operations, or verify the equivalence of representations indicates an inadequate environment for the cultivation of reversible thinking skills. The tasks are meticulously structured to prioritize forward thinking, wherein the solution technique is explicitly delineated, enabling students to follow the procedure without the need to reconstruct the mathematical reasoning underlying it. This design has been developed to mitigate the likelihood of students engaging in backward thinking, re-evaluating strategies, or verifying the consistency of results through the inverse process. These elements are central to reversible thinking, which has been demonstrated to influence the strengthening of students' concepts related to the topic of comparison (Saparwadi et al., 2020).

This reversible thinking strategy is seldom discussed (Ramful, 2014), primarily due to the limited research conducted on it. Another reason is that it is a component of higher-order thinking processes, where students must conceptualize the assimilation of parts into a whole and perceive the whole as a unified entity composed of parts. However, students of formal age should already possess this ability. At this stage, children can enhance their capacity for reversible thinking. According to Piaget, reversible thinking commences during the concrete operational stage, spanning the ages of seven to eleven (Lamon, 2007). Students who develop reversible thinking strategies can effectively solve problems.

CONCLUSIONS

The findings of this study indicate that the comparison tasks included within the Grade 7 Mathematics textbook for junior high school Semester 2 do not effectively promote the development of students' reversible thinking abilities. A praxeological analysis of the tasks, techniques, and technology reveals that nearly all tasks are procedural and forward-thinking oriented, with explicit solution steps provided. This limitation restricts students' opportunities to explore concepts independently. While certain tasks may inherently support two-way reasoning, such as comparing quantities with different units, the task design does not offer opportunities for reversal activities, equality checks, or backtracking. These activities are crucial for facilitating reversible thinking. These findings emphasize the necessity of enhancing the design of mathematics textbook tasks, making them more challenging, open-ended, and conceptual. This enhancement is essential in fostering the development of higher-level mathematical thinking skills in students.

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